Chapter 2

Optimal Pensions in Segmented Financial Markets

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Abstract

Low interest rates, low growth and population ageing put existing pension systems under pressure. The outbreak of the Great Recession pushed some governments to adopt reforms in order to contain the costs of social security and restore its financial sustainability. However, the disruption of the financial system and its consequences for households' finances pose new challenges for policy-makers. I develop a quantitative overlapping generations general equilibrium model with heterogenous agents to examine the role of a specific financial friction hindering access to capital markets in the design of optimal pension systems. I show that the welfare-maximizing level of social security crucially depends on the size of the financial market imperfection.

Advisors: Davide Debortoli, Jordi Galí

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1 Introduction

Reforming social security is a pressing matter in many countries in the western world. While a prolonged period of low interest rates and low growth has dramatically reduced the returns of existing public pension systems, an inexorable population ageing process threatens their financial sustainability. Several economies have adopted counter measures such as increasing the retirement age to rebalance the dependency ratio between a shrinking working age population and an expanding mass of retirees or modifying the way the first pension is calculated for a given amount of contributions [6]. Some governments, in the attempt of reducing the liabilities of increasingly more costly and underperforming pension systems, have introduced a multi-pillar structure containing a funded defined-contribution component. Although the literature on pensions has extensively discussed the costs and benefits of social security and how it is affected by demographic dynamics, limited attention has been devoted to the impact of financial frictions on household finances and the corresponding optimal response of social security.

This paper studies the optimal design of pensions under a specific set of assumptions, aimed at capturing the current economic outlook and, in particular, the distortions pertaining to financial markets as a consequence of the Great Recession and the secular decline in interest rates. Specifically, I introduce a novel imperfection, referred to as *minimum investment requirement*, limiting access to capital markets in a medium-scale overlapping generations model. The presence of heterogenous agents and idiosyncratic shocks generates an endogenous wealth distribution and a differential ability across households to transfer consumption over time. Such a setup provides a motive for having pensions as a means of insurance policy and intergenerational risk-sharing.

The first piece of empirical evidence motivating this project resides in the increasing trend of the equity risk premium observed over the last decade, documented by Caballero & Farhi (2017) [5]. As Figure 1 illustrates, already before the onset of the financial crisis, global markets experienced a secular decline in the interest rates. Still, the risk-free and equity return moved together. Starting from 2007-08, however, there has been a decoupling of the two measures, suggesting a surge of the equity risk premium. The proponents of the secular stagnation hypothesis argue that this empirical fact is due to a shortage of safe assets, monetary policy decisions, a productivity slowdown and population ageing.

The second stylized fact supporting this paper is the extremely limited asset market participation of lower wealth classes, as reported in Figure 1 from Mian and Sufi's blog "House of Debt" in 2014 and based on US Survey Consumer Finance data.

These two empirical facts corroborate Piketty's (2013) thesis of an increasing inequality within society, if only the agents at the top of the wealth distribution can access the riskier capital returns. This economic scenario is the one that I attempt to replicate in the macroeconomic model examined in this paper. Without investigating further on its determinants (which is beyond the scope of this paper) I will assume that a minimum investment requirement restricts participation to capital market only to those households who have accumulated enough wealth, thanks to inheritance or earnings over the lifetime. The conjecture that higher wealth is associated with higher returns to savings is crucial to justify the optimality of a public pension system. Fagereng, Guiso, Malacrino & Pistaferri (2016) [10] show that this assumption is a realistic one using a rich dataset compiled from Norway's administrative tax records. Furthermore, Fagereng, Gottlieb & Guiso (2017) [9]convey that both capital market participation and the share invested in risky equity display a hump-shaped pattern over the lifetime, which is an implication of assuming a minimum investment requirement.

This analysis is connected with several strands of literature. Similarly to Krueger & Ludwig (2016) [15], I examine the use of fiscal policy in the presence of heterogenous agents and uninsurable idiosyncratic earnings risk in an OLG setting. Nonetheless, rather than studying the right mix of education subsidies and labor income taxes, I focus on the optimal design of pension systems. This projects contributes to the literature on the optimal size of social security and the comparison between pay-as-you-go and fully-funded systems (Matsen & Thøgersen (2004) [16], Miles (2000) [17], Krueger & Kubler (2006)[14], A. Imrohoroglu , S. Imrohoroglu & Joines (2003) [13]). Differently from the existing literature, the model developed here investigates the role played by limited asset market participation of heterogenous households in defining the optimal pension policy. The assumptions of a minimum investment requirement in the capital market, bequest motives and labor earnings risk links this paper with the line of research on the long-term determinants of the wealth distribution and consumption inequality over the life-cycle (Benhabib, Bisin & Zhu (2011) [4], Benhabib, Bisin & Luo (2015) [3], Storesletten, Telmer & Yaron (2004) [18], Huggett, Ventura & Yaron (2011) [12], De Nardi & Fang (2014) [7]). The economic environment depicted by the framework analyzed in this paper shares common traits with the one studied in the secular stagnation hypothesis literature (Caballero & Farhi (2017), Caballero, Farhi & Gourinchas (2017) [5], Eggertsson & Mehrotra (2014) [8], (2017)). Finally, the macroeconomic model proposed in this work follows the tradition of employing overlapping generations models for studying fiscal policy pioneered by Auerbach & Kotlikoff (1987) [2] and relies on idiosyncratic shocks to labor income as in Bewley (1986), Huggett (1993) and Aiyagari (1994) [1].

The paper proceeds as follows: section 2 introduces the minimum investment requirement in a simple 2-period model; section 3 illustrates the key ingredients of the heterogenous agents OLG model adopted in this paper, section 4 presents the results for the optimal design of social security and section 5 concludes.

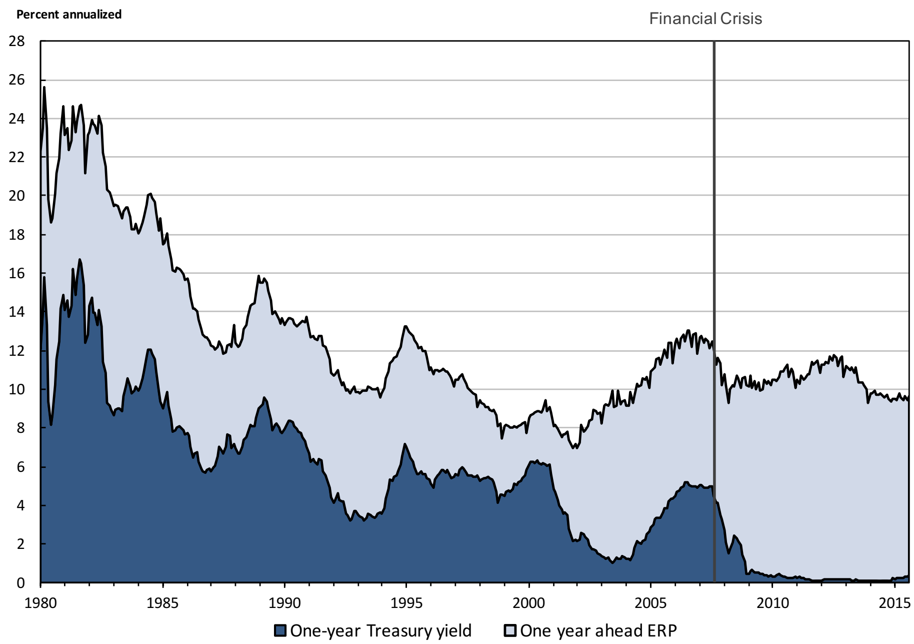

Figure 1: Equity Risk Premium over time, from Caballero & Farhi's "The Safety Trap" (2017)

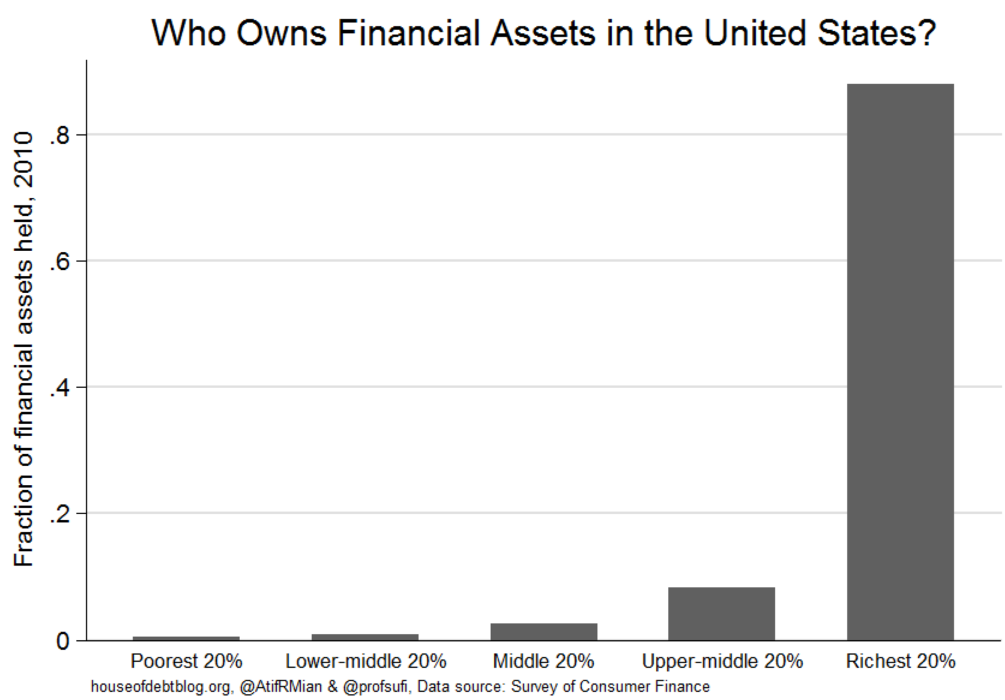

Figure 2: Asset ownership per income per quantile, from Mian & Sufi's "House of Debt" blog (2014)

2 Minimum investment requirement

The purpose of this section is to illustrate the main market imperfection studied in this paper at work in a stylized 2-period representative agent model. In order to simplify the analysis, I will set up an endowment economy where the return on savings is exogenously fixed. Endogenous production will be introduced in the next section, where the prices of labor and capital will be determined in a general equilibrium framework.

In this simplified world, the economy exists for two periods. In the first period $t = 0$ the representative household is born with the exogenous endowment W and decides what portion of it to consume and what portion to save for the next period $t = 1$ in a standard lifetime utility maximization problem. The amount that is saved can be invested in two different opportunities, both having an exogenous deterministic return. The first investment opportunity is a basic storage technology, with return $r^l = 0$; the second one is an asset with return $r^h > 0$. Furthermore, the latter can be purchased only with a minimum investment $I \geq \bar{I}$. This minimum investment requirement is a financial friction that hinders the access of the consumer to the asset market. Formally, the consumer problem is:

$$\max_{C_0^Y, C_1^O, I} \quad \log(C_0^Y) + \beta \log(C_1^O)$$

$$\text{s.t.} \qquad C_0^Y + I = W$$

$$C_1^O = (1 + r)I$$

$$r = \begin{cases} r^h & \text{if } I \geq \bar{I} \\ r^l & \text{if } I < \bar{I} \end{cases}$$

Obviously, in this stylized economy, the presence of a representative household implies that either no savings or all savings are invested in the high return investment opportunity, depending on the threshold level $\bar{I}$.

The solution of the optimization problem entails applying the standard Kuhn-Tucker conditions and identifying three different regions: $I > \bar{I}$, $I = \bar{I}$ and $I < \bar{I}$:

When the minimum investment requirement $\bar{I}$ is relatively small, the household's consumption and investment decisions are not affected by its presence. Therefore, as it follows from log utility, she will

	$I > \bar{I}$	$I = \bar{I}$	$I < \bar{I}$
C_0^Y	$\frac{1}{1+\beta}W$	$W - \bar{I}$	$\frac{1}{1+\beta}W$
C_1^O	$\frac{\beta}{1+\beta}W(1+r^h)$	$\bar{I}(1+r^h)$	$\frac{\beta}{1+\beta}W(1+r^l)$
I_t	$\frac{\beta}{1+\beta}W$	$\bar{I}$	$\frac{\beta}{1+\beta}W$

Table 1: Three different regions

invest a fraction $\frac{\beta}{1+\beta}$ of her initial endowment W entirely in the asset and benefit in period $t = 1$ from the high market return r^h.

As the threshold level $\bar{I}$ is increased, we reach the second region. The minimum investment requirement is now binding and it is optimal for the household to invest exactly $I = \bar{I}$ because of the higher return. This implies that the first period consumption C_0^Y is going to be lower than in the previous region, as the investment requirement is higher, but second period consumption C_1^0 will be higher.

If $\bar{I}$ is increased further, we end up in the third region, where it is not optimal anymore to invest in the asset as the minimum requirement is too high and all the savings are invested in the low-yield storage technology with return $r^l = 0$. Again, as a consequence of log utility, the amount saved will not depend on the return r^l, but will be the same fraction of the endowment W as in the first region.

Figure 3 summarizes these results, assuming $W = 0.5$, $r^l = 0$, $r^h = 0.3$, $\beta = 0.9$ and varying $\bar{I}$ from 0.1, to 0.3 and finally to 0.4, in order to highlight the three different regions.

In this otherwise standard intertemporal decision problem, the minimum investment requirement in the asset market is graphically represented by a jump in the budget line, where the change in the slope is due to the different returns offered by the two investment opportunities. The horizontal distance between the point of discontinuity and the first period endowment W measures $\bar{I}$, the size of the friction.

A higher $\bar{I}$ reduces the consumption set of the household by increasing the minimum investment necessary to access the high market return r^h. The consumer household will postpone consumption and save a greater portion of her endowment W than the one she would save in absence of such market imperfection, but only as long as $\log(W - \bar{I}) + \beta \log((1 + r^h)\bar{I}) \geq \log(\frac{1}{1+\beta}W) + \beta \log(\frac{\beta}{1+\beta}W)$, i.e. as long $\bar{I}$ is such that the economy is in the second region.

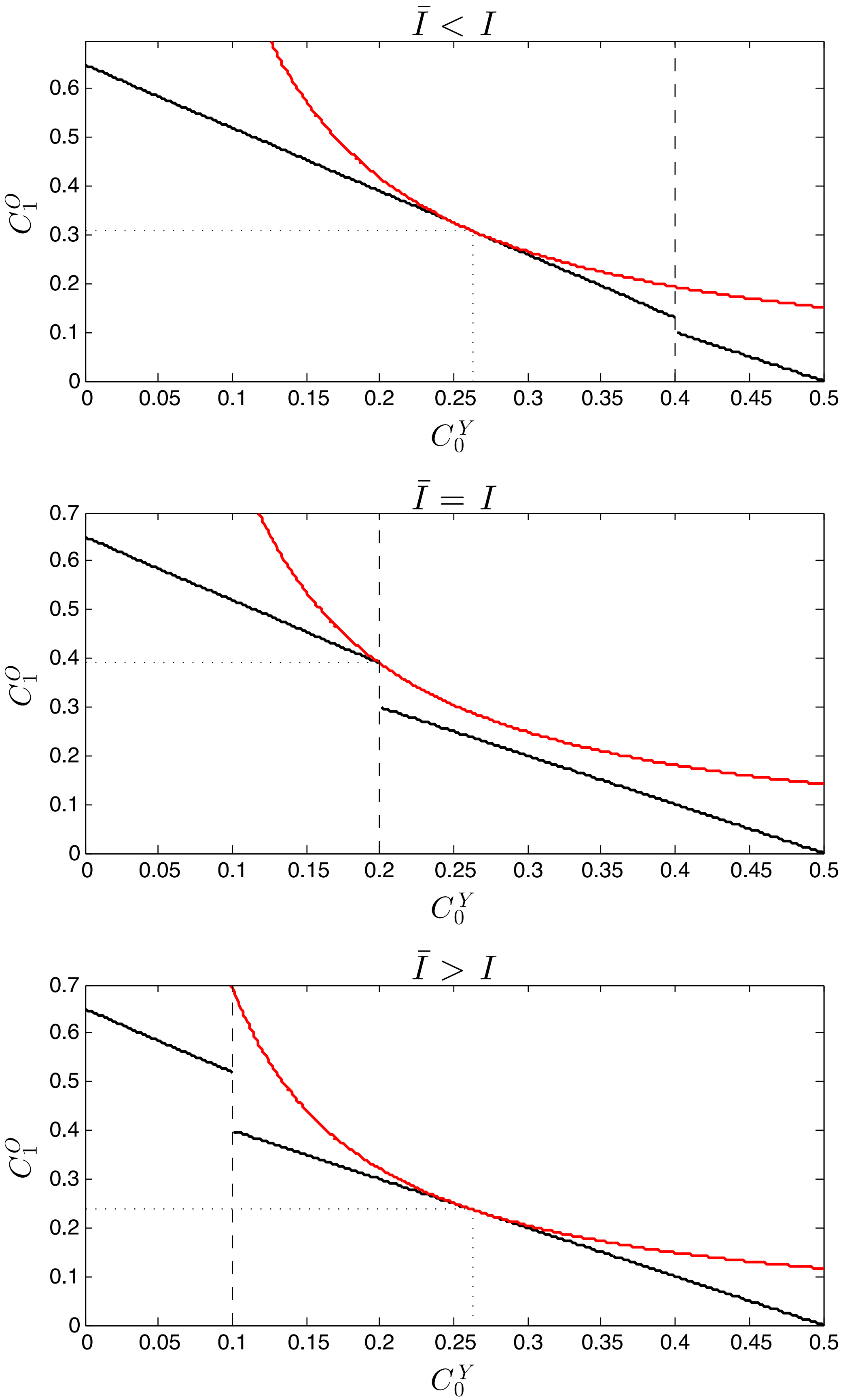

Figure 3: Consumption/Savings decision for different levels of $\bar{I}$

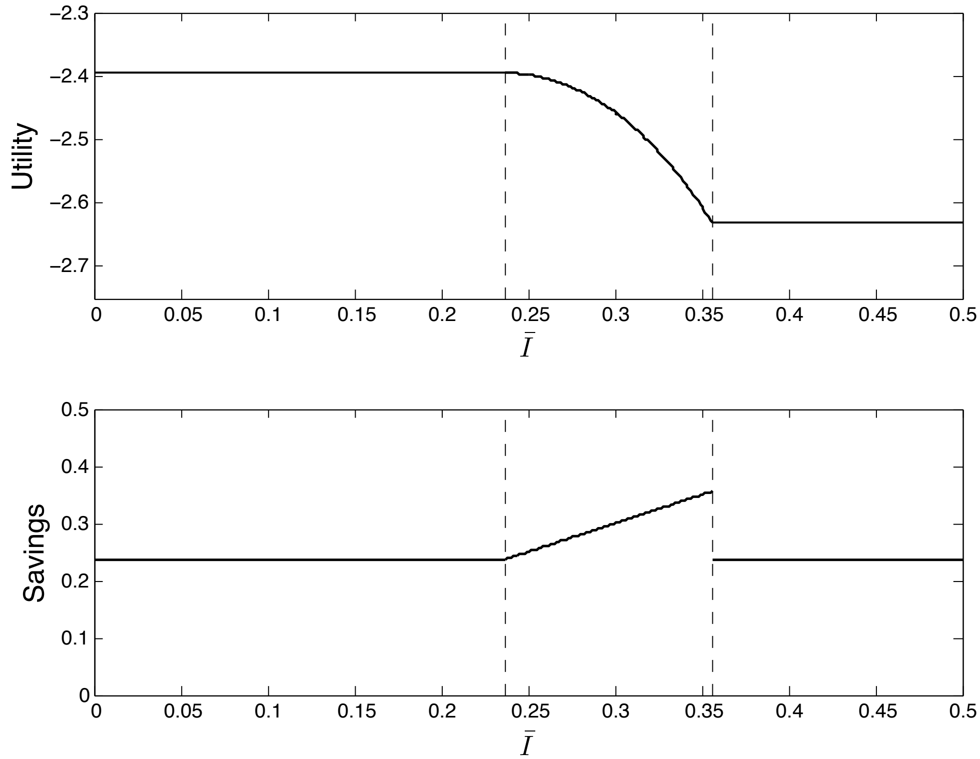

Figure 4: Utility and Savings for different levels of $\bar{I}$

Figure 2 shows optimal savings and the corresponding utility for different levels of the threshold $\bar{I}$. Again, for relatively low levels of the minimum investment requirement, the investment decision is not affected. After the first cutoff level, higher $\bar{I}$ implies higher optimal savings, while lifetime utility falls. When $\bar{I}$ is higher than the second cutoff level, optimal savings plummets to the levels of the first region because it is not convenient any longer for the household to sacrifice further first period consumption to access the asset market.

The role played by this market imperfection is interesting: it generates a non-monotone response of aggregate savings to changes in the minimum investment requirement. Therefore, in a more complicated version of this model, business cycle dynamics could emerge as the result of consecutive shocks to $\bar{I}$, gradually increasing aggregate investment and then suddenly making it drop. Besides its business cycle dimension, examining this financial friction and its determinants seems a promising research prospect for several reasons. Firstly, the presence of a minimum investment requirement could contribute to quantitatively account for the sources of inequality along the business cycle. Secondly, an increase in its size can potentially explain also the secular decline in interest rates documented in the secular stagnation hypothesis literature. In fact, a bigger barrier to private financial markets pushes consumers either to invest more, and therefore increase the demand for assets and their prices, or to allocate their savings in storage technologies with a lower return, as simple bank accounts, determining an overall fall in the aggregate real interest rate, computed as the weighted average return of savings.

The presence of a minimum investment requirement limiting participation to financial markets creates scope for public intervention. A welfare improvement is achieved if the government creates an asset, like a mandatory public pension system, which is not affected by any entry barrier and whose return is higher than r^l. The objective of this paper is to find the optimal level of social security as a function of the size of the financial friction described in this section. In the stylized model here presented, the existence of a representative household makes this task relatively easy and of limited interest: the result obtained in partial equilibrium analysis would be hardly extendible to more realistic setups. The research question examined in this paper is a quantitative one and, as such, it requires a set of assumptions able to replicate what is observed in the data. In particular, the design of pension systems should credibly follow existing schemes.

Modelling a realistic pension system however, brings about further distortions and complications. The first typical distinction is between pay-as-you-go and fully funded systems (respectively PAYG and FF in the rest of the paper), depending on the way the pension arrangement is funded and on the determination of its return. Further levels of characterization involve establishing whether pensions are defined-benefit or defined-contribution, the provision of a minimum pension, the strength of the linkage between individual contributions and benefits, indexation to wages or prices, life-expectancy automatic adjustments, multi-pillar designs, the formula adopted to calculate benefits, the pool and risk profile of assets available for purchase to the public pension funds, the degree of intergenerational and intragenerational risk sharing.

The literature on pensions has extensively discussed the costs and benefits of different ingredients shaping pension schemes and these aspects go beyond the scope of this analysis. Still, little attention has been devoted to the impact of restricted market participation on the design and size of social security. With this purpose in mind, the natural way to proceed is to abandon the assumption of a representative household and to examine the implications of having heterogenous agents. In particular, generating a credible wealth distribution constitutes a necessary step in order to study how different agents, who find themselves in different parts of the wealth distribution, are affected by the minimum investment required to access the financial market and its higher returns. As a consequence, the optimal level of social security will be the one maximizing a social welfare function, weighing the individual utilities of heterogenous agents. The next section introduces the overlapping generations model with heterogenous consumers which will be the workhorse model for our quantitative exercises.

3 General Equilibrium Model

The natural framework for studying problems dealing with pensions and ageing is overlapping generation models. In the related literature, two are the options usually considered. On the one hand, the Blanchard-Yaari-Gertler perpetual youth model offers a relatively tractable environment, where the problem of aggregation is easily solved using the law of large numbers. Nonetheless, the assumption of age-independent probabilities of retirement and death represents a limitation for exercises that quantitatively examine consumers' choice along the life-cycle. For this reason, our choice falls on models a-là Auerbach-Kotlikoff, characterized by a full life-cycle structure with multiple generations co-exisiting at the same time and age-dependent survival probabilities. This class of models is more computationally demanding, but allows for more realistic demographic dynamics.

The overlapping generation model introduced in this section differs from the partial equilibrium framework described in the previous section in several aspects. The most relevant one concerns the determination of the interest rates. Specifically, while in the previously presented stylized economy both the storage technology return and the asset return were fixed parameters, here I will assume that only the former is exogenous and that the high market return is the return to capital, determined in equilibrium by demand and supply. This has a major implication: a change in the minimum investment requirement will affect the supply of capital, and, therefore, its return.

Furthermore endogenous production replaces the exogenous endowment and the demand side, rather than being summarized by the behavior of a representative consumer, will be populated by heterogenous households.

As the purpose of this paper is to determine the long-term optimal size of social security, the analysis will focus on steady state exercises and leave aside transition dynamics.

3.1 Heterogenous households

Households have identical preferences, but differ in terms of wealth, ability and age. The life-cycle structure is as follows: households enter the economy at age 25 with an initial inherited stock of assets and an initial skill endowment, stay in the labor force until age 65 when they deterministically retire. After retirement, they live off of their asset returns and pension benefits until a maximum age of 100. The age of death is stochastic and the survival probabilities are age-dependent. Between 25 and 65, workers gain higher productivity through experience. Stochastic death and a bequest motive imply that households perish with positive assets which are automatically transferred, as inheritance, to the new generation of households replacing them in the next period. The stochastic age of death triggers the precautionary motive for saving. Such assumptions, together with the differential stock of wealth and skill at birth, will generate both a wealth and an income distribution. An endogenously generated wealth distribution will affect the ability of heterogenous agents to access the financial markets. The minimum investment requirement illustrated in the previous section, in combination with a borrowing limit, completes the set of constraints. The household entering the economy at time t and aged 25 faces the following lifetime utility maximization problem:

$$\max_{c_{t+k-1}(i),a_{t+k}(i)} E_t \sum_{k=1}^{75} \beta^{k-1} \left[\prod_{z=1}^{k} \left(1-\theta_{z-1}\right) \right] \left[U\big(c_{t+k-1}(i)\big) + \frac{1}{\beta}\theta_{t+k} W\big(a_{t+k+1}(i)\big) \right]$$

s.t.

- if young and employed:

$$c_{t+k}(i) + a_{t+k+1}(i) = a_{t+k}(i)\big(1 + r_{t+k} - \delta\big) + (1-\tau_{t+k}) w_{t+k} l_{t+k} e_{t+1}(i,k) \quad \forall \quad 0 \leq k \leq 39$$

- if old and retired:

$$c_{t+k}(i) + a_{t+k+1}(i) = a_{t+k}(i)\big(1 + r_{t+k} - \delta\big) + pen_{t+k}(i,k) \quad \forall k \geq 40$$

- borrowing constraint:

$$a_{t+k}(i) \geq 0 \quad \forall k$$

- minimum investment requirement:

$$r_{t+k} = \begin{cases} r^h_{t+k} & \text{if } a_{t+k}(i) \geq \bar{a} \\ r^l_{t+k} & \text{if } a_{t+k}(i) < \bar{a} \end{cases} \qquad \forall j$$

- initial asset endowment:

$$a_t(i,0) = a^0(i)$$

The function U is a standard CRRA: $U\big(c_{t+k}(i)\big) = \frac{c_{t+k}(i)^{1-\sigma}}{1-\sigma}$ and expresses the utility coming from consumption. The function $W\big(a_{t+k+1}(i)\big) = \chi\frac{a_{t+k+1}(i)^{1-\eta}}{1-\eta}$ represents the utility derived from leaving bequests and takes as argument the amount of assets at the age of death. The temporal budget constraint at time $t+k$ depends on the state of the household i in the same period: employed or retired. It is assumed that, when young, each consumer inelastically supplies a unit of labor $l_{t+k} = 1$, augmented by the individual productivity or ability level $e_{t+k}(i,k)$. After the exogenously fixed retirement age of 65, i.e. 40 years after entering the labor markets, the households starts receiving a pension benefit $pen_{t+k}(i,k)$ until death. Uncertainty stems from one source only and only at an individual level: life duration. The stochastic process describing it is characterized by the sequence $\theta_0, ..., \theta_{75}$ of age-dependent probabilities of dying, with $\theta_0 = 0$ and $\theta_{75} = 1$.

At this point, we can rewrite the household problem more conveniently using two sets of Bellman equations, one valid as long as the household is aged between 25 and 65 and one valid after retirement. For $1 \leq k \leq 40$:

$$V\big(a_t(i); e_t(i,k); k\big) = \max_{c_t(i);a_{t+1}(i)} U(c_t(i)) + \theta_k W\big(a_{t+1}(i)\big) + \beta(1-\theta_k)V\big(a_{t+1}(i); e_{t+1}(i,kk+1); k+1\big)$$

subject to

$$c_t(i) + a_{t+1}(i) = a_t(i)\big(1 + r_t - \delta\big) + (1-\tau_t)w_t e_t(i,k)$$

For $k > 40$:

$$V\big(a_t(i); k\big) = \max_{c_t(i);a_{t+1}(i)} U(c_t(i)) + \theta_k W\big(a_{t+1}(i)\big) + \beta(1-\theta_k)V\big(a_{t+1}(i); k+1\big)$$

subject to

$$c_t(i) + a_{t+1}(i) = a_t(i)\big(1 + r_t - \delta\big) + pen_t(i,k)$$

The minimum investment requirement and borrowing constraint apply too.

3.2 Supply side

The production sector of the economy is standard. There is a representative firm hiring capital and labor in perfectly competitive markets and employing a constant returns to scale Cobb-Douglas production function $Y_t = A_t K_t^{\alpha}(E_t L_t)^{1-\alpha}$ where A_t represents an aggregate level of productivity growing at the exogenous and deterministic rate g and E_t the aggregate labor augmenting productivity which is assumed to be constant over time. As a consequence, production factors are paid their marginal productivities:

$$r_t^K = \alpha A_t \left(\frac{K_t}{E_t L_t}\right)^{\alpha-1}$$

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{E_t L_t}\right)^{\alpha} E_t$$

3.3 Government

The government collects a labor income tax, whose revenues are used to finance the pension transfers and runs a balanced budget in every period. Two alternative pension systems are considered.

Pay-as-you-go

The PAYG scheme is a simple transfer from the working households to the ones retired. The tax rate on labor income is adjusted to satisfy two requirements at the same time. Firstly, a certain replacement ratio ϕ_t is ensured, so that pensioners receive a fraction of the average wage they earned during their working age. It is, therefore, a defined benefit scheme. Secondly, the tax rate is set to preserve the financial sustainability of social security in each period.

$$\underbrace{\sum_{k=1}^{40}\sum_{i} N(i,k)\tau_t^{PAYG} w_t e_t(i,k)}_{\text{total revenues}} = \underbrace{\sum_{k=41}^{75} N(i,k) pen_t(i,k)^{PAYG}}_{\text{total expenditures}} \tag{1}$$

where

$$pen_t(i,k)^{PAYG} = \phi_t \frac{1}{40}\sum_{j=1}^{40}(1-\tau_{t-k+j}^{PAYG}) w_{t-k+j} e_{t-k+j}(i,k-40+j) \tag{2}$$

Equation 1 represents the government balanced budget, where total labor income tax revenues equate total expenditures for pensions. $N(i,k)$ is the mass of individual i at age k and $e_t(i,k)$ is her ability level at time t. The term $\sum_{k=39}^{74} N(i,k) pen_t(i,k)^{PAYG}$ constitutes total expenditure for pension benefits, paid to all retired households at time t, and crucially depends on the size of each cohort born at time $t-k$ and still alive at time t, $N_{i,k}$. Finally, equation 2 shows how an individual PAYG pension is calculated: it is a fraction $0 < \phi_t < 1$ of the average after tax labor income of agent i during her working age (25-65).

After fixing the replacement ratios $\phi_t = \phi$ the tax rate τ_t^{PAYG} is adjusted so that equation 1 - 2 are satisfied:

$$\tau_t^{PAYG} = \frac{\frac{\phi}{40}\sum_{k=41}^{75} N(i,k)\sum_{j=1}^{40}(1-\tau_{t-k+j}^{PAYG}) w_{t-k+j} e_{t-k+j}(i,k-40+j)}{\sum_{k=1}^{40}\sum_i N(i,k) w_t e_t(i,k)} \tag{3}$$

Which states that, in a PAYG system, the tax rate is increasing in the defined benefits due to the pensioners, determined by the retired cohorts' size $\{N(i,k)\}_{k=41}^{75}$, the average after tax wage they received during working age$\frac{1}{40}\sum_{j=1}^{40}(1-\tau_{t-k+j}^{PAYG}) w_{t-k+j} e_{t-k+j}(i,k-40+j)$, the replacement ratio ϕ, and decreasing in aggregate labor income $\sum_{k=1}^{40}\sum_i N(i,k) w_t e_t(i,k)$.

As the last equation shows, PAYG schemes are exposed to some risks. In particular, population ageing, by reducing the size of young cohorts relative to the old ones, puts into question the sustainability of this arrangement. As population ages, higher tax rates are required to ensure a balanced budget, if the government aims at maintaining the desired replacement ratio ϕ. This demographic effect is countered by wage growth: if the economy grows the upward pressure on the tax rate is relieved. In particular, the rate of return of a PAYG pension system is strictly correlated with the return of the economy.

Fully-funded

A FF arrangement is funded in a completely different way: individual contributions are collected into a collective fund investing in some asset or financial opportunity, and subsequently distributed as annuities after retirement. The return of the fund is, in substance, the return of the portfolio chosen by the fund managers. In our model, the FF system invests in the only available financial asset, capital. Whether or not this corresponds to a realistic assumption is open to discussion. Under the regulations of many countries the pension funds of FF systems can invest only in a restricted class of assets, strictly defined in terms of risk-return profiles and issuing institution. The ability of FF schemes to diversify efficiently their portfolios and ensure a minimum return represents, in reality, one of their limitations. As a matter of fact, the risks connected to the return are practically borne by the pensioners and there is no intergenerational risk sharing.

In our framework, the value of the individual fund for the household i at retirement age (65) is equal to the sum of total capitalized contributions:

$$\text{Value Fund}_{t,40}(i,40)^{FF} = \sum_{j=0}^{40} \tau_{t-j}^{FF} w_{t-j} e_{t-j}(i,k-j)(1+r_{t-j}^{K})^{j} \tag{4}$$

From the first year of retirement, the pension fund starts paying an annuity, but what is left in the fund keeps generating interests. Therefore, the yearly pension benefit is calculated as follows:

$$pen_t^{FF}(i) = \frac{\text{VF}_{t,40}(i,40)^{FF} E_t \prod_{j=0}^{35}(1+r_{t+j}^{K})}{\sum \prod_{k=}(1+r_{t-j}^{K})^{j}} \tag{5}$$

The FF system depicted in this framework is relatively convenient: intragenerational risk sharing allows pensioners to insure themselves against the demographic risk due to the uncertainty on the age of death. Moreover, this pension arrangement offers the high market return, the capital return, without facing the minimum investment requirement, which households would encounter privately.

In the literature on pensions, it is often argued that, transition costs aside and in absence of other types of frictions, it is preferable to move towards fully funded systems, whose return, the real interest rate or return to capital, has been historically higher than the economy growth rate. In fact, in dynamically efficient economies, for those agents who have access to capital markets, a PAYG scheme reduces lifetime income: the resources collected by the government will yield less than the financial markets, determining a loss. Instead, in dynamically inefficient economies, i.e. those economies where $r^K < n+g$, it is possible to produce a Pareto improvement by setting up a transfer from younger to older generations and solving the problem of over-accumulation of capital. In this paper, we study the effect of the secular decline in the interest rate and the corresponding increase in equity risk premium on optimal pension design. As a consequence, we will examine the desirability of alternative pension arrangements and discuss the well-known comparison between PAYG and FF systems in light of the developments taking place in the current demographic and economic scenario.

3.4 Competitive equilibrium

The competitive equilibrium of this economy is defined as:

- a series for the individual productivities $\{e_t(i,k)\} \quad \forall i,k$;
- a series of allocations $\{c_t(i), a_{t+1}(i)\}_{t=0}^{\infty} \quad \forall i$;
- a series of factor prices $\{w_t, r_t^K\}_{t=0}^{\infty}$;
- a series of tax rates $\{\tau_t\}_{t=0}^{\infty}$.

such that firms maximize their profits, households maximize their lifetime utility and the government runs a balanced budget in every period, ensuring the desired replacement ratios.

As the objective of this paper is to study optimal pensions in the presence of a minimum investment requirement limiting access to financial markets, we need to solve a Ramsey social planner problem. It consists of determining the tax rate series $\{\tau_t\}_{t=0}^{\infty}$ maximizing a utilitarian social welfare function that is compatible with a competitive equilibrium allocation, i.e. such that the first order conditions describing the optimal behaviour of firms and households are satisfied, and the government budget is balanced.

$$\max_{\{\tau_t\}_{t=0}^{\infty}} \sum_i \frac{N(i,k)}{\text{life exp}(k)} \sum_{j=1}^{75-k} \beta^{j-1} \left[\prod_{z=k}^{k+j} \left(1-\theta_{z-1}\right) \right] \left[U\left(c_{t+j-1}(i,k+j-1)\right) + \frac{1}{\beta}\theta_{k+j-1} W\left(a_{t+j}(i,k+j-1)\right) \right]$$

with $\theta_{k-1} = 0$ and where $N(i,k)$ indicates the mass of agent i at age k. The utilitarian social welfare function is the average of all existing households' expected lifetime utilities weighted by their size relative to the overall population and their life expectancy. The solution of the Ramsey social planner's problem, as the one for the competitive equilibrium, entails dealing with a large system of non-linear equations that cannot be solved analytically. Hence, I resolve to standard numerical methods to derive the stationary distribution.

How is the optimal level of social security determined? In order to answer this question, it is necessary to analyze the costs and benefits of pensions in relation with the assumptions made in this model. Obviously, the preference over the tax rate depends on the state, defined as a combination of age, productivity and financial position, featured by the individual agent.

On the side of costs, a higher tax rate means, for workers, a lower disposable income and therefore a crowding out effect on private accumulation of savings. Agents who cannot access the financial markets will sacrifice a greater portion of their after-tax labor income for savings, in the attempt of satisfying the minimum investment requirement in the following periods. Pensions are an asset whose return is accessible only once retired. Differently from the case of private savings, if a worker is hit by a negative income shock she cannot run down her pensions entitlements to smooth her consumption. In practice, pensions are illiquid until retirement age.

Secondly, depending on the specific arrangement (in our analysis PAYG or FF), pension systems can have a lower return than the private return to savings, especially for those households being able to access the capital market. The return of the PAYG scheme, for example, is the growth rate of the economy. Forcing agents to contribute to a PAYG system when the return to capital is higher than the growth rate of the economy implies a reduction of the agent's lifetime income. The size of this crowding out effect is increasing in the difference $r^K - (n+g)$ and in the tax rate τ. However, the quantitative importance of these distortions caused by financing social security crucially depends, in this model, on the barrier limiting access to the return to capital. The example presented in the previous section showed that a household that is unable, independently from social security, to accumulate enough savings to satisfy the minimum investment requirement over her lifetime will not be affected by the crowding out effect. As a matter of fact, if her private savings return is r^l, the basic storage technology return and the return of the pension asset is higher than that, a positive tax rate will increase her lifetime income. This suggests that in this model a pension system represents an opportunity for some households and an impediment for others: the individual current and future expected asset position, relative to the minimum investment requirement, determines the desirability of social security as a means for transferring value into the future and smooth consumption.

In addition, a pension arrangement serves another purpose: it acts as an insurance policy against demographic risk. By pooling together the mandatory savings of all agents, social security ensures old-age consumption independently of the individual lifetime duration. As a consequence, in a FF system agents living longer/shorter than life expectancy e_{40} at the age of retirement are the winners/losers of this government provision as they receive as pension payments more/less than they contributed, while in a PAYG scheme a defined benefit is granted independently of the total amount of individual contributions.

As a final note, the assumption of exogenous labor supply means that taxes on labor income will not distort the typical consumption/leisure trade-off.

3.5 Calibration

Table 2 summarizes the calibration of most parameters. Most of it is due to the calibration/estimation performed in Benhabib, Bisin & Luo (2015). Although the model laid out in this article slightly diverges from theirs, especially in relation with the presence of the minimum investment requirement and heterogeneous agents within each cohort, this analysis closely follows their approach to derive a realistic wealth distribution. An important result of their exercise is that $\sigma > \eta$, i.e. the utility from consumption displays higher curvature than the one obtained from bequests. This translates into increasing saving rates with respect to wealth. As explained by the authors, this is one of the determinants of the concentration in wealth observed in reality. A second crucial factor is labor earnings risk. In this regard, we refer to the work by Heathcote, Perri & Violante (2010) [11], which documents, based on PSID data, the evolution of the labor earnings distribution along the life-cycle. It uncovers the increasing dispersion of labor earnings over age and a typical hump-shaped profile. Table 3 presents their findings. Ten different labor earnings classes are identified featuring heterogenous initial labor proceedings and heterogenous labor income growth rates, which are accounted for in the model. As the data for earnings close to retirement age are unavailable, I assume that the observed fall in earnings between age 50 and 55 takes place in a larger time span, until 65. Moreover, as shown in the first column of Table 3, the bottom 10% of the earnings distribution features negative earnings, a symptom of debt positions. Consistently with Benhabib, Bisin & Luo (2015), negative earnings are replaced by a fictitious tiny value as one of the model assumption is a no borrowing constraint. The presence of a bequests motive generates a distribution of wealth at birth. To keep track of inheritance and early life asset positions I distinguish

Parameter	Value	Description
α	0.33	capital share in production
β	0.97	discount rate
σ	2	relative risk-aversion coefficient
δ	0.1	depreciation rate
η	1.186	curvature parameter for utility from bequests
χ	0.0312	weight of bequests in the utility function
r^l	0	storage technology return
A	1	total factor productivity
g	0	technological progress

Table 2: Calibration

Age range/%	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
25-30	-2.68	9.356	16.87	23.23	29.47	35.48	41.71	49.12	59.52	87.90
31-36	-1.68	12.90	21.88	29.78	37.10	44.21	52.06	61.69	75.01	123.5
37-42	-1.73	13.48	23.84	32.88	41.35	49.64	57.95	68.42	84.67	153.8
43-48	-2.73	13.59	24.54	33.73	42.76	51.46	60.73	72.46	90.04	165.5
49-54	-4.97	10.47	20.95	29.68	38.81	47.98	57.98	69.65	87.23	165.2
55-60	-8.22	10.47	11.31	19.63	28.21	37.60	47.20	59.23	77.07	156.5

Table 3: Labor earnings (in thousand dollars) profiles by age range from PSID and cleaned by Heathcote, Perri & Violante (2010)

5 different classes at birth, based on the quintiles of the endogenous bequests distribution. However, to simplify the analysis, I will assume that labor earnings and initial asset positions groups are orthogonal, i.e. being born with a low or high initial wealth endowment does not influence the probability of being part of a specific labor earnings class. Therefore, the model does not account for human capital accumulation and education investment.

The data source for the age-dependent survival probabilities for the US is the 2014 Actuarial Life Table, provided by the US Social Security Administration. I assume that each period the size of the newborn cohort, entering the economy, is exactly 1. As both fertility and death rates are constant over time, population is also constant. Figure 3.5 shows the weight of each age cohort in the overall population. The area underneath the curve amounts to life expectancy at birth.

Finally, the benchmark model will feature 10 different households (each one accounting for 10% of the population) characterized by distinct labor earnings profiles and 5 different wealth positions at birth, determined by the quantiles of the endogenous bequests distribution. Thus, 50 heterogenous agents coexist in each cohort so that the model keeps track of 50x75 agents of different mass.

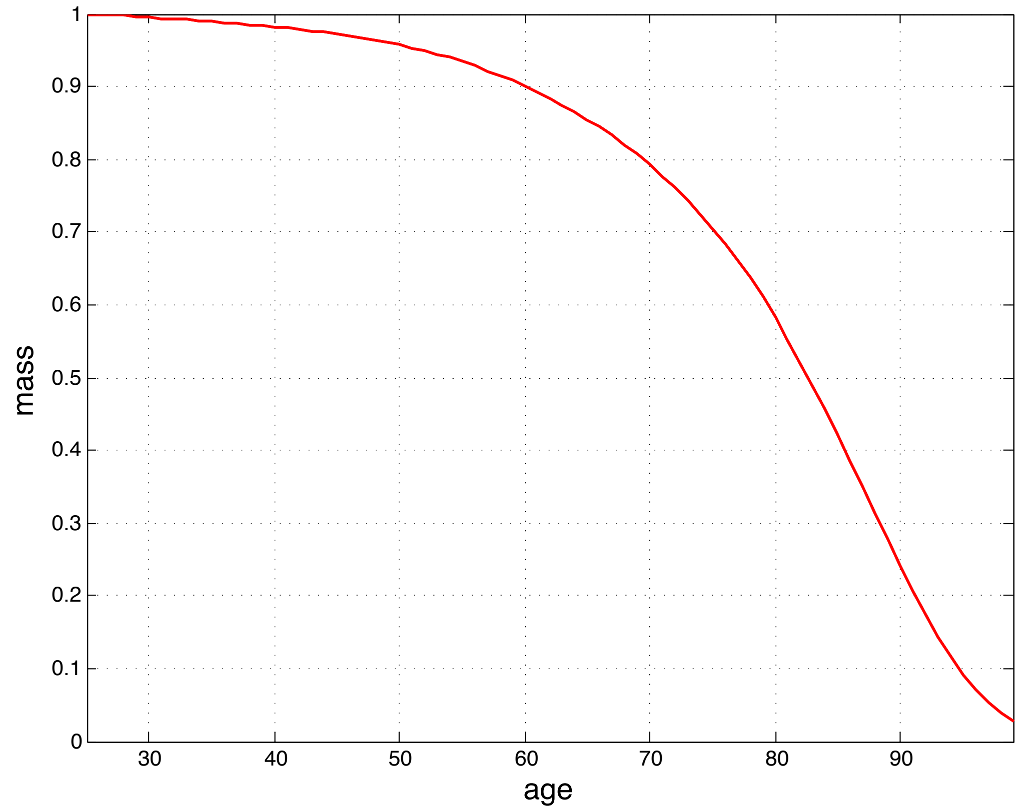

Figure 5: Population mass by cohort

4 Results

4.1 The benchmark economy

The first experiment I conduct consists of examining how the equilibrium stationary distribution is affected by different levels of the minimum investment requirement in absence of social security. In particular, I will assess the impact of this friction on a set of variables, namely the return to capital, the average return to savings of the economy, aggregate output, the cross-sectional consumption inequality over the lifetime through simple comparative statics exercises. The objective of this analysis is twofold. First of all, to check whether the model assumptions are able to replicate some empirical facts that are central for the purposes of this paper. For example, it is important to verify the fit of the wealth distribution endogenously derived in this model. Secondly, to determine the scope of this market imperfection, or, in other words, whether the presence of even a small minimum investment requirement is quantitatively important for the equilibrium properties of the economy under examination.

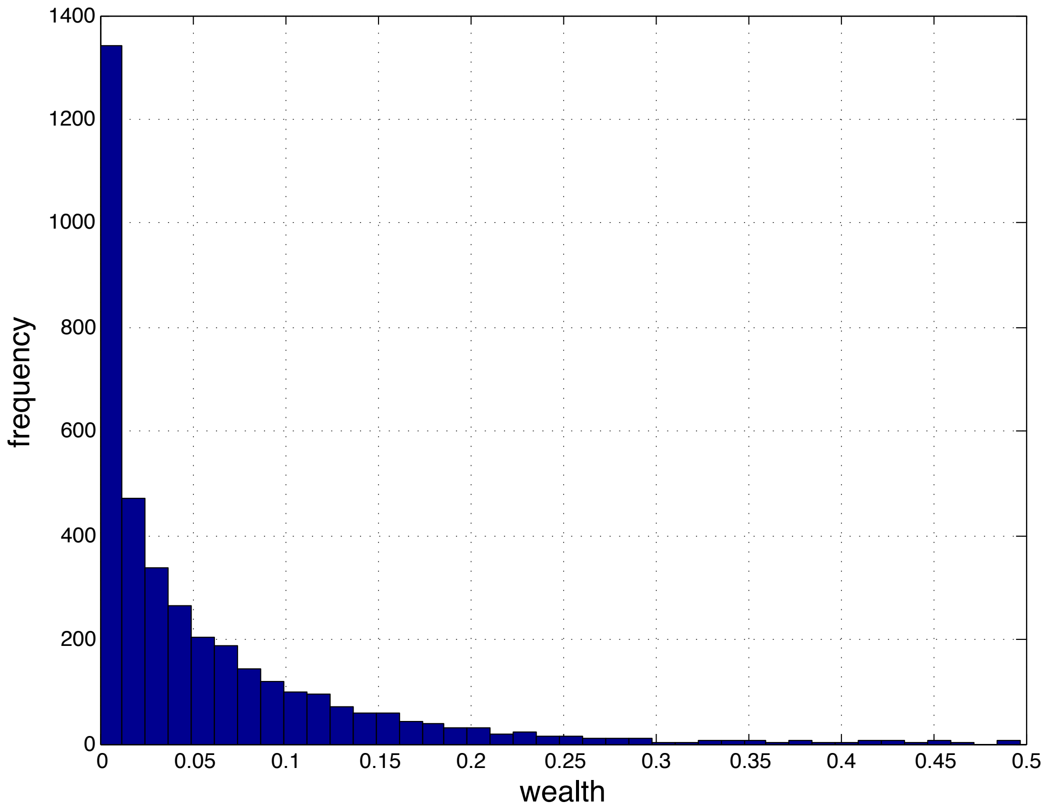

Figure 6: Wealth distribution when $\tau, \bar{a} = 0$

Variable	$\bar{a} = 0$	$\bar{a} = 1$	$\bar{a} = 2$	$\bar{a} = 3$	$\bar{a} = 5$
w	0.5585	0.5568	0.5425	0.5173	0.4499
r^K	0.0612	0.0616	0.0649	0.0715	0.0950
K	3.1269	3.0981	2.8633	2.4785	1.6230
$\bar{r}$	0.0612	0.0397	0.0310	0.0250	0.0173
S	3.1285	3.2070	3.1792	3.0486	2.6386
Y	0.5801	0.5783	0.5635	0.5373	0.4673
Gini	0.6238	0.6219	0.6308	0.6441	0.6666

Table 4: Comparative statics with $\tau = 0$

Figure 4.1 displays the wealth distribution generated by the model, in absence of any pension system and of any minimum investment requirement. Not surprisingly, it closely resembles the one obtained in Benhabib, Bisin & Luo (2015) and therefore matches quite well US data. Table 4 summarizes the results of this first exercise, where the minimum investment requirement is increased from 0 to 5. A larger threshold $\bar{a}$ implies that households will be able to access the capital market and its higher return only at later stage of their working life, after stockpiling enough savings. When the minimum investment requirement is high, the relatively low-skilled agents are completely excluded from the capital market and only the low-yield storing technology is available to them. As a consequence, the amount of capital supplied to firms falls and so do production and wages, according to the neo-classical production function. The equilibrium between demand and supply in the competitive capital market implies that return to capital r^K increases as $\bar{a}$ rises, while the average interest rate or return to savings falls. Interestingly, both aggregate savings S and the Gini coefficient for wealth dispersion initially increase when the minimum investment requirement is increased from 0 to 1. This can be interpreted as the optimal response of agents who, facing a higher barrier to the capital, are willing to adjust their savings in order to access the higher asset return r^K. However, when $\bar{a}$ is increased further, aggregate savings drop and wealth inequality increases.

Figures 4.7 and 4.8 show the effect of increasing $\bar{a}$ from 0 to 2 on the cross-sectional consumption inequality along the life-cycle. Each line represents the path of consumption of one of the 10 different labor earnings classes (the bottom 10% us barely visible). For this comparison, initial wealth positions determined by bequests do not matter as the 5 different wealth classes at birth are aggregated. Therefore, all labor earnings classes start their life with the same amount of assets. Their ability to reach the minimum investment requirement is uniquely determined by the evolution of their labor income over the working life. As it emerges in Figure 4.1, households cross $\bar{a}$ threshold either twice or never during their lifetime. The bottom 20% of the labor earnings distribution never save enough to access the capital return r^K, when $\bar{a} = 2$. The other classes access the capital market at different age in their youth and exit at the end of their life. Intuitively, a faster productivity growth allows individuals to reach the threshold earlier. The graph in 4.1 displays an additional feature: for most classes able to access the capital return at some point in their life, the consumption profile shows a well defined U-shaped pattern. Initially, in order to save enough, households reduce their consumption over time. Once reached an amount of savings larger than $\bar{a}$, their consumption profile starts to increase with age, but only until the retirement age. From that moment on, due to the fall in yearly income (labor earnings plummet to zero), agents start running down their private assets and consume only out of their capital income. If we compare this patterns with the ones emerging from Figure 4.1, where $\bar{a}$ is set to zero, it is evident that in the latter consumption inequality is uniquely determined by the different individual productivity growth rates, while, in presence of a barrier to the capital market, the timing of entry of the different labor earning classes plays a role in shaping cross-sectional consumption inequality over the life-cycle. Storesletten, Telmer & Yaron (2004) document an increase in consumption inequality with age and attribute this phenomenon to labor earnings risk. Huggett, Ventura & Yaron (2011) instead claim that the increasing consumption inequality over the lifetime can be mostly accounted for the initial conditions, defined in terms of asset position, skill level and learning ability. The existence of a minimum investment requirement entailing a restricted capital market participation can bridge these two opposing views.

Figures 4.1 and 4.1 display how the savings of the different income groups evolve over the life-cycle and how they are affected by the presence of the market imperfection introduced in this paper. Consistently with standard life-cycle models, agents accumulate savings during their working age and start dissaving after retirement. Moreover, Figure 4.1 shows that different households cross the $\bar{a}$ threshold at different age: richer households are able to access the capital market at an earlier stage of life and exit from it later, just few years before the maximum age agents can reach (100).

Overall, the outcome of this first exercise uncovers the potential of the minimum investment requirement market imperfection analyzed in this paper to explain some of the stylized facts presented in the introduction. In particular, it can contribute to account for the secular decline in interest rates, the increase in equity risk-premium, the historical increase in wealth inequality and the observed consumption increasing dispersion along the life-cycle.

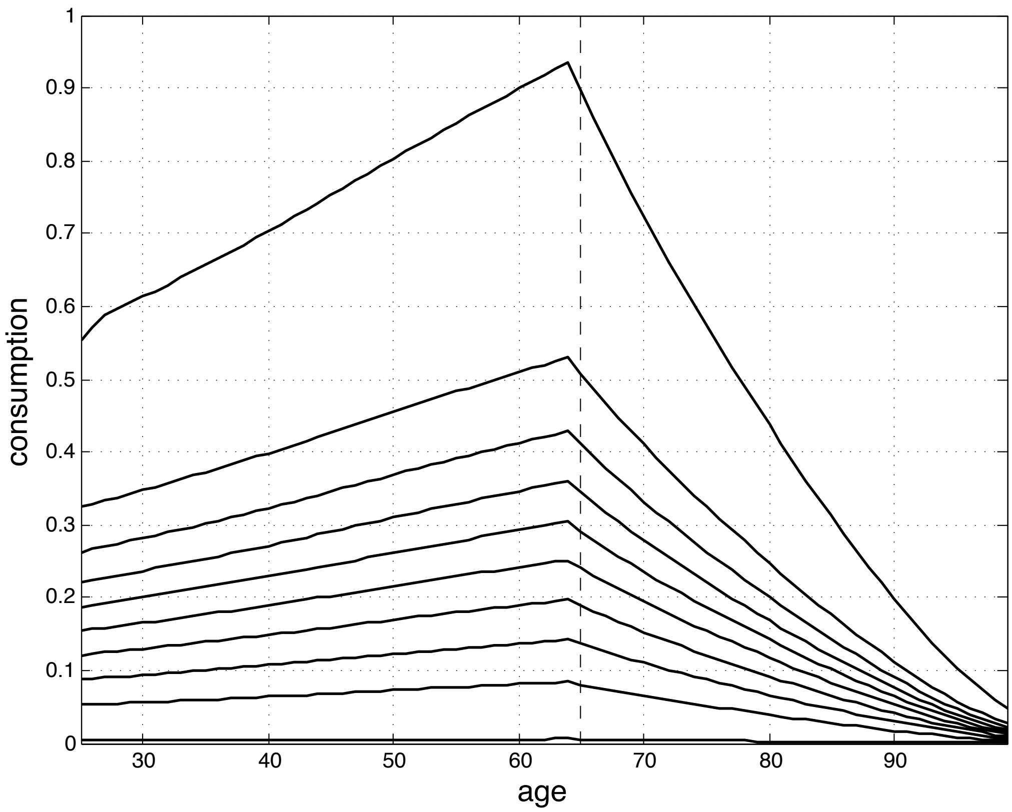

Figure 7: Cross-sectional consumption inequality over lifetime with $\bar{a} = 0$ and $\tau = 0$

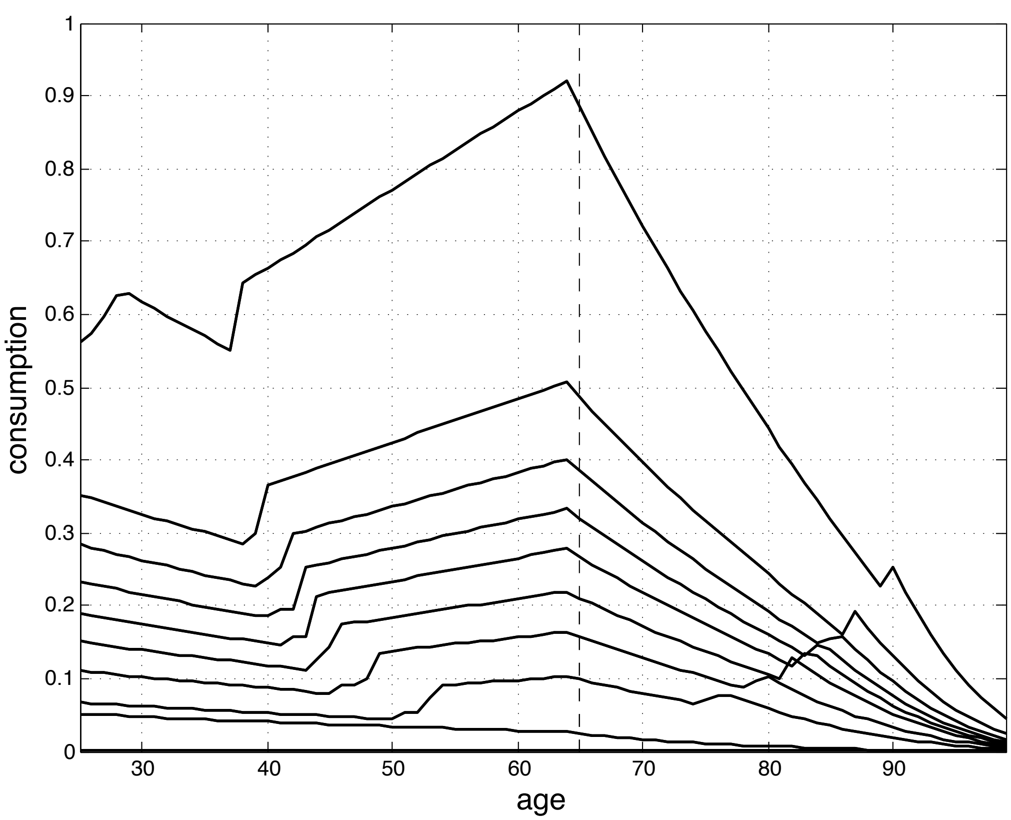

Figure 8: Cross-sectional consumption inequality over lifetime with $\bar{a} = 2$ and $\tau = 0$

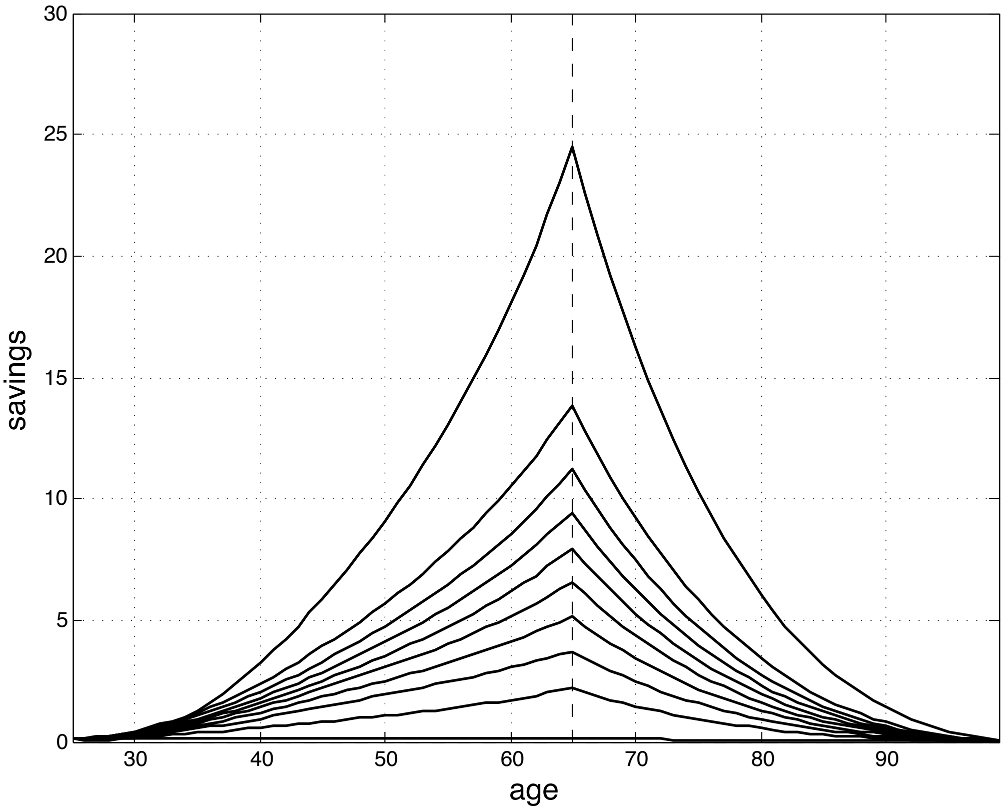

Figure 9: Cross-sectional savings inequality over lifetime when $\bar{a} = 0$ and $\tau = 0$

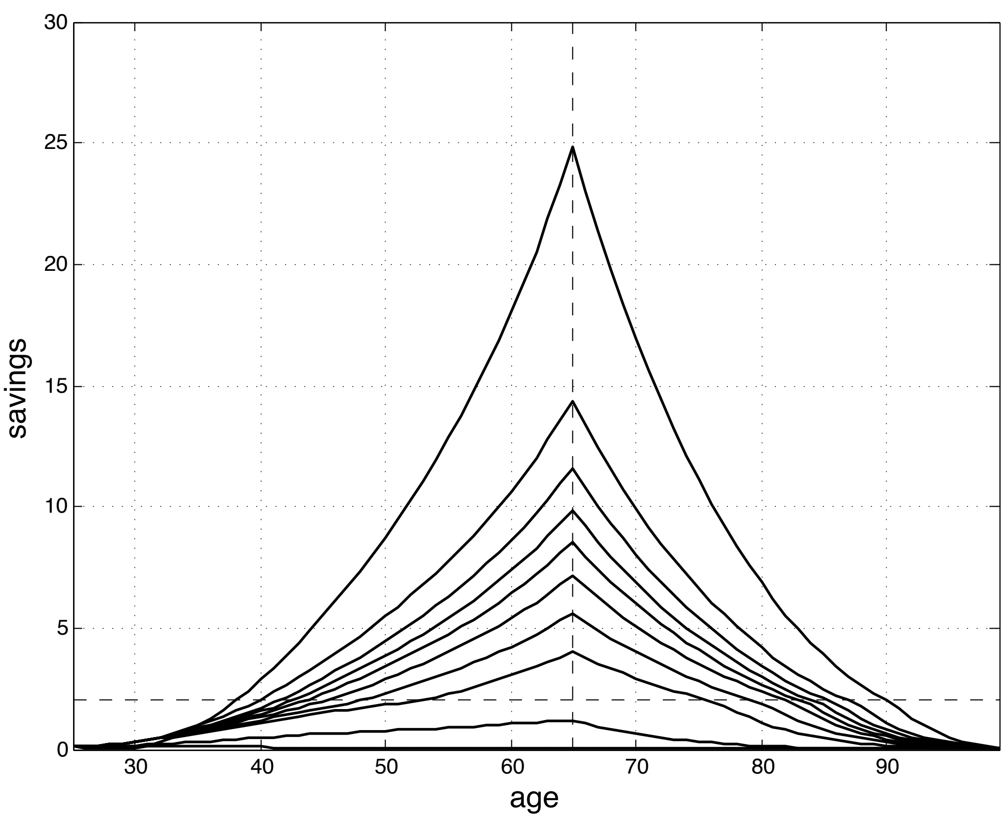

Figure 10: Cross-sectional savings inequality over lifetime when $\bar{a} = 2$ and $\tau = 0$

4.2 Pay-as-you-go

This sub-section derives the optimal size of a PAYG pension system in the benchmark economy described above. I obtain the tax rate maximizing the social welfare under two different scenarios, when $\bar{a}$ is 0 and when it is 2. In the first case, all households are able to individually access the capital market and its higher return r^K. In the second case, when $\tau = 0$, the presence of a minimum investment requirement completely precludes the participation of the bottom 20% of the labor earnings distribution (the two lowest classes) to the capital market, while all the other households manage, each at a different age, to accumulate enough wealth to pass the threshold.

The main feature of this type of scheme is the redistribution of all the collected contributions it entails. In practice, each labor earnings class and each cohort contributes the same proportion of her labor income, but a different amount in absolute terms due to heterogenous productivity levels across skill and age groups. However, each retiree, independently of total individual contributions, receives the same pension benefit. As a consequence, even though the aggregate return of the PAYG pension arrangement is the economy growth rate, the individual return to the pension asset varies across labor earnings groups: it will be high for the households at the low-end of the earnings distribution and low for top-earners. Interestingly, in presence of a minimum investment requirement in the capital market, the return of a PAYG scheme will be especially high relative to the return to private savings for the poorest households. At the same time, a positive tax rate τ will crowd out private savings and make the $\bar{a}$ threshold harder to reach, hurting the lifetime utility of the wealthier households. Therefore, this specific form of social security promotes risk-sharing both across households belonging to different income groups and across different generations. Tables 5 and 6 report the equilibrium summary statistics for different tax rates, when $\bar{a} = 0$ and when $\bar{a} = 2$, respectively.

When $\bar{a} = 2$ the optimal tax rate τ is around 0.65%. As Figures 4.2 and 4.2 show, the introduction of a PAYG system pushes up the lifetime utility profile of the bottom 20% of the earnings distribution, corresponding, in the graphs, to the two lowest curves. The lifetime utility of the other households is almost unaffected by the positive tax rate, despite the increase in r^K and the decline in w.

Variable	$\phi = 0.01$	$\phi = 0.015$	$\phi = 0.02$
τ	0.0044	0.0065	0.0087
w	0.5540	0.5519	0.5498
r^K	0.0622	0.0627	0.0632
K	3.0513	3.0157	2.9819
$\bar{r}$	0.0622	0.0627	0.0632
S	3.0528	3.0173	2.9834
Y	0.5754	0.5732	0.5711
SWF	-224.0521	-208.9430	-222.8154
Gini	0.6289	0.6311	0.6332

Table 5: Comparative statics with $\bar{a} = 0$ and PAYG pension system

Variable	$\phi = 0.01$	$\phi = 0.015$	$\phi = 0.02$	$\phi = 0.025$	$\phi = 0.05$	$\phi = 0.075$
τ	0.0044	0.0065	0.0087	0.0108	0.0214	0.0317
w	0.5376	0.5348	0.5308	0.5270	0.5160	0.5042
r^K	0.0662	0.0669	0.0679	0.0689	0.0719	0.0754
K	2.7853	2.7419	2.6796	2.6222	2.4596	2.2926
$\bar{r}$	0.0310	0.0310	0.0305	0.0300	0.0298	0.0302
S	3.0817	3.0354	2.9658	2.8981	2.7159	2.5425
Y	0.5583	0.5555	0.5513	0.5473	0.5359	0.5236
SWF	-320.2069	-311.8015	-315.7505	-320.0942	-323.9293	-339.6559
Gini	0.6418	0.6449	0.6551	0.6657	0.6835	0.6916

Table 6: Comparative statics with $\bar{a} = 2$ and PAYG pension system

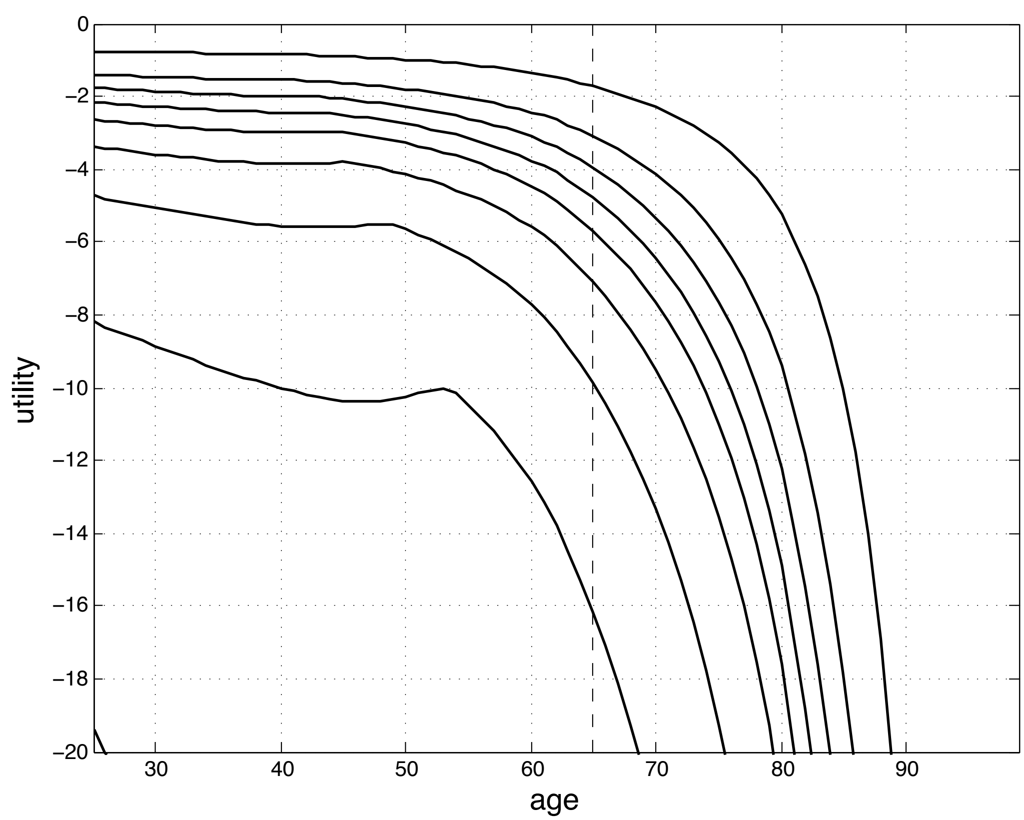

Figure 11: Cross-sectional lifetime utility inequality over lifetime with $\bar{a} = 2$ and $\tau = 0$

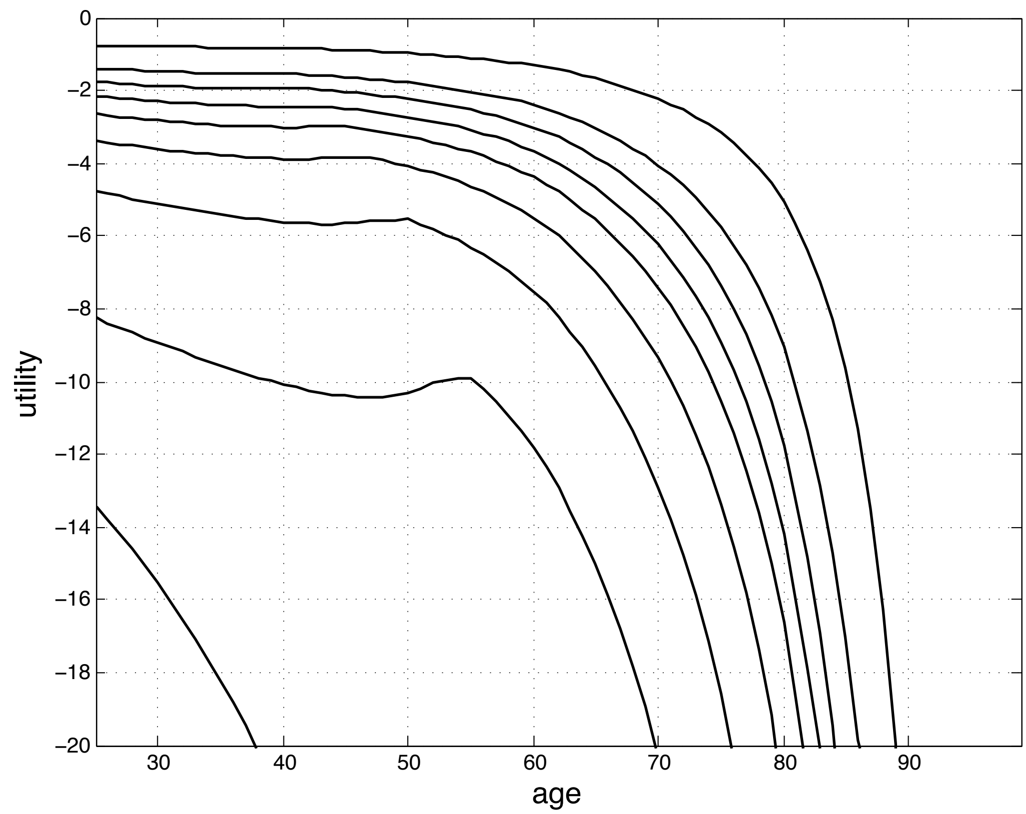

Figure 12: Cross-sectional lifetime utility inequality over lifetime with $\bar{a} = 2$ and $\tau = 0.65\%$

4.3 Fully funded

The final exercise of this paper is to determine the optimal of size of a fully funded pension system in the modelled economy. The presence of this type of scheme produces some positive and some negative effects. Specifically, by pooling resources together, it allows households to insure against demographic risk and to benefit from the high market return r^K. It does so by reducing the disposable labor income during working age and crowding out private savings. Moreover, some households benefit and some others lose from it.

Within each labor earnings class, those who die before life expectancy at the age of retirement end up contributing more to the fund than the cumulative amount that is paid to them in terms of annuities. In addition, differently from private savings, the remaining resources in the pension fund at the age of death cannot be recovered as bequests, but are used to pay the pension benefits of those who live longer than life expectancy at the age of retirement.

Across different labor earnings classes, benefiting the most from this type of pension arrangement are those that would never accumulate enough savings to reach the minimum investment requirement in

Variable	$\tau = 0.05$	$\tau = 0.1$	$\tau = 0.15$	$\tau = 0.2$	$\tau = 0.25$	$\tau = 0.3$
w	0.6199	0.6818	0.7370	0.7880	0.8373	0.8858
r^K	0.0495	0.0408	0.0349	0.0304	0.0269	0.0240
K	4.2878	5.7221	7.2441	8.8735	10.6642	12.6491
$\bar{r}$	0.0225	0.0173	0.0137	0.0105	0.0084	0.0068
S	2.8169	2.4945	2.1889	1.8842	1.6261	1.4372
Y	0.6438	0.7081	0.7654	0.8184	0.8696	0.9200
SWF	-3.8326e+04	-2.4267e+04	-1.8887e+04	-1.6074e+04	-1.3703e+04	-1.1996e+04
Gini	0.6495	0.6636	0.6737	0.6919	0.7002	0.7080

Table 7: Comparative statics with $\bar{a} = 2$ and FF pension system

absence of social security due to low initial ability and limited productivity growth. A fully funded pension arrangement allows these agents to enjoy a higher return on their contributions to the pension fund than the one received on their private savings. Furthermore, a larger τ implies that more resources on aggregate are invested in capital through the pension fund, increasing aggregate output and wages and decreasing r^K. At the same time, workers' private savings are crowded out due to a reduction in disposable income. The net effect of these two opposing forces favours, for the chosen calibration, an increase in total capital as a result of a stronger income effect ($\sigma > 1$), inelastic labor supply and the presence of the minimum investment requirement. The latter makes the consumption/savings decision of the poorest households almost insensitive to changes in the return to capital. Exactly for this reason, they benefit the most from a positive tax rate as it allows them to enjoy a higher wage while being only partially affected by the lower return to capital (through the pension annuities). All other households endure a utility loss when social security gets larger as a lower disposable income makes the minimum investment requirement harder to reach. In particular, the earnings class facing the largest disadvantage when the tax rate is high is the one whose productivity evolution along the life-cycle is such that its labor proceedings are just enough to pass the $\bar{a}$ threshold at some point in their life when $\tau = 0$. Therefore, if the tax rate is too high, this class will be excluded by the capital market. Table 7 and Figures (())) highlight these results.

5 Conclusions

This project investigates the role played by a minimum investment requirement to access the capital market in the design of the optimal pension system. It develops an overlapping generation model, in the tradition of Auerbach & Kotlikoff, populated by heterogenous households to study distributional issues. In order to derive wealth and income distributions able to realistically mimic the US data, the model features a bequest motive and heterogenous labor earnings as proposed by Benhabib, Bisin & Luo (2015). Agents, at birth, differ along two different dimensions: labor earnings path and initial wealth. To avoid further complications (and in absence of a reliable calibration) it is assumed that the skill level and the evolution of labor income over the life-cycle are independent of the initial stock of assets. Therefore, each of the different wealth classes at birth faces the same probability to end up in one of the ten labor earnings groups. The existence of an entry barrier to financial markets pushes some agents to adopt a low-yield storing technology to transfer value over time, smooth consumption and save against demographic risk. Social security, depending on the specific pension scheme implemented, offers an opportunity to insure agents against longevity uncertainty, to promote some type of risk-sharing across heterogenous households and to limit the social cost of the minimum investment requirement. In particular, a PAYG system favours inter-generational redistribution as part of the pension benefits of low-income retirees are paid by high-income workers but reduces the lifetime income of the richer households by crowding out their private savings. A FF pension asset instead grants the higher return r^K, some degree of intra-generational risk-sharing within each household type, but offers no inter-generational risk-sharing as pension benefits are perfectly linked to individual contributions.

Two are the main results of this analysis: the introduction of both pension systems increases the long-term social welfare function and the optimal size of social security crucially depends on the threshold $\bar{a}$. Specifically, optimal pension policy hinges on the relative densities of the households accumulating an amount wealth just below and above the minimum investment requirement.

Future developments of this line of research will involve measuring the size of this market imperfection in the data and for different economies and micro-founding it. However some objectives seem more urgent at this stage. Firstly, the model lacks a realistic correlation between asset positions at birth and labor earnings evolution over the life-cycle. So far, this correlation is assumed to be zero. Secondly, the two schemes here examined prove to be an important test for the analysis of the minimum investment requirement financial friction, but fall short in realistically match existing pension systems. The PAYG arrangement features an artificially loose connection between individual contributions and benefits and an exaggerated degree of redistribution. The FF plan discussed in the model invests in the only available asset, capital, and therefore is unable to capture the complex problems faced by real pension funds in their portfolio allocation decision.

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