The Optimal Taxation of Risky Entrepreneurial Income

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Abstract

I analyse optimal fiscal policies that balance insurance, redistribution and efficiency in an incomplete markets framework with uninsurable idiosyncratic entrepreneurial risks. I show that the optimal capital income tax rate crucially depends on two elasticities: (1) the elasticity of capital investment with respect to the tax rate and (2) the elasticity of the equilibrium wage with respect to aggregate capital. Policy makers need to take this second elasticity into account because the affect of capital investment on equilibrium wages is not internalized when markets are incomplete.

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1 Introduction

Entrepreneurship is one of the main drivers of wealth inequality. In particular, successful entrepreneurs contribute substantially to the observed concentration of wealth in the developed world. Cagetti and DeNardi (2006) document that in the United States the share of entrepreneurs in the top one percentile of the wealth distribution is five to seven times larger than their share in the overall population. While, depending on the exact definition of entrepreneurship, only between 7.6% and 16.7% of the total population are entrepreneurs, these individuals hold between 33% and 53% of total US wealth.[1]

The optimal taxation of these individuals is a reoccurring theme in the political debate. Typically, entrepreneurs become rich for a combination of, at least, two reasons: ability and luck.[2] Proponents of a higher tax rate thus often argue that some of the most well of in a society - be it through inherited ability or luck - should contribute more. In this way, not only a certain amount of redistribution (from able to unable) is achieved but also insurance against adverse idiosyncratic shocks. On the other hand, opponents of higher taxes on entrepreneurs argue that these individuals provide jobs to a large share of the population. Taxes on entrepreneurial income are distortive, reducing incentives to invest and hence labor demand. Thus high tax rates on entrepreneurs would in the end harm the the working poor, who might have difficulties finding jobs or experience cuts in their wages.

The analysis of these arguments is at the center of the present paper. To be specific, I derive optimal tax formulas in an incomplete markets model, where entrepreneurial investment is risky. These formulas optimally balance redistribution, insurance and efficiency. I show that the optimal linear tax on capital income crucially depends on two elasticities: (1) the elasticity of capital investment with respect to the tax rate and (2) the elasticity of the equilibrium wage with respect to aggregate capital. The first elasticity is rather standard. A higher tax rate typically leads to a reduction in taxable income, the tax base. The government thus needs to balance the direct effect of higher tax revenue per dollar of income with the indirect effect of lower gross (taxable) income. The second elasticity is not standard and a consequence of the incomplete markets assumption. It turns out that in this case the laissez-faire economy suffers from a severe form of inefficiency: A welfare improvement could be achieved would each agent simply invest a bit more than he/she does in the laissez-faire equilibrium, i.e. without any ex-post redistribution of income. This form of inefficiency, sometimes referred to as "constrained inefficiency" has recently been analysed by Davila, Hong, Krusell, and

[1] Cagetti and DeNardi (2006) document these figures on the basis of the 1989 Survey of Consumer Finances (SCF). Similar figures are documented in Quadrini (2000).

[2] Of course, with overlapping generations another reason is that businesses are inherited. In the infinite horizon setting of the present paper, the heir of a successful business and its inheritor are considered as the same economic agent.

Rios-Rull (2012) in the context of idiosyncratic labor income risk and by Gottardi, Kaji, and Nakajima (2016) in a more general setting with both idiosyncratic labor- and capital income risk. Households do not internalize the effect of their investment decisions on prices. When labor and capital are complements a higher capital stock increases the income per unit of labor (wages), while decreasing the income per unit of capital invested. In the present framework the former is riskless and the latter is risky. Hence, a planner who is prevented from engaging in any form of direct insurance or redistribution, but is only allowed to dictate the households' savings decision while satisfying their budget constraints, would choose higher capital investment than in the laissez-faire allocation. This way she achieves indirect insurance (by upscaling riskless wage income and downscaling risky capital income) and indirect redistribution (by upscaling the part of the income received by all agents as opposed to the income received mostly by the rich), simply by affecting prices. I show that this effect, often referred to as "pecuniary externality", is quantitatively important, substantially reducing the optimal tax rate on capital income, in fact, potentially even changing its sign.

Analyzing optimal taxation in a model with idiosyncratic investment risk is not just a pure theoretical exercise. The presence of such risks is widely documented in the empirical literature. For example, Moskowitz and Vissing-Jorgenson (2002) find that about 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth and that households with entrepreneurial equity invest on average more than 70 percent of their private holdings in a single private company for which they have an active management interest. It has further been shown that these risks are one of the main drivers of wealth inequality (Piketty (2014), Benhabib, Bisin, and Zhu (2011)).[3]

Motivated by these facts a large body of theoretical literature emerged that tries to analyze the effect of this type of risk on macroeconomic aggregates, in particular capital accumulation. Early contributions along these lines include Covas (2006) and Angeletos (2007). Both authors compare the equilibrium capital stock to the one obtained under complete markets and conclude that, depending on the calibration, both under- or overinvestment of capital compared to complete markets benchmark is possible. Such a comparison has, however, little to say whether, and to what extent, capital should be taxed as it compares two different environments with two different market structures. Instead, this paper affiliates to the literature on optimal fiscal policies when capital income is risky and uninsurable.

The remainder of the paper is organized as follows. In the next section 2 I present the main mechanisms by means of a simple two period model. This section is divided in three subsections. In the first subsection, I derive the laissez-faire equilibrium, in

[3] Aside from entrepreneurship, the focus of this paper, it has been shown that these returns can result from different positions in agents' financial portfolios. For example, some households invest only in riskless bonds, while others also hold stocks, which - while being risky - deliver higher returns on average (Guvenen (2006), Guvenen (2009)).

the second subsection the constrained efficient allocation and in the third subsection the optimal Ramsey plan for a planner who has access to linear capital income taxes and anonymous lump sum transfers. In section 3 I extend the analysis to an infinite horizon. This section is work in progress and hence not yet included in the paper. The plan is to calibrate this model to the US economy in order to obtain quantitative results for the optimal level of capital income taxes. Finally, in consider the appendix I consider model extensions that will eventually enter the quantitative version of the model (financial intermediation and a corporate sector).

2 Two Period Model

The main mechanisms can be explained by means of a simple two period model. There is a continuum of agents of measure one, who value consumption in both periods. The expected lifetime utility of each agent is given by

$$u(c_0) + \beta \mathbb{E} u(c_1),$$

where $u(.)$ is increasing, concave and satisfies typical Inada conditions: $u'(c) > 0$, $u''(c) < 0$, $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$.

Each agent runs a firm. Initially, in period $t = 0$, agents are endowed with a certain amount of the only consumption good. We will use the terms endowments and assets interchangeably. Assets a are distributed over agents according to some distribution function $G(a)$. In the initial period agents decide on the fraction of assets they consume and save but there is no production. In the second period, $t = 1$, savings (capital) are used together with labor input for production and output is consumed.

The output of each firm is produced according to a constant returns to scale neoclassical production function that is capital augmenting in entrepreneurial ability

$$y = F(zk, l),$$

where k and l are the production inputs capital and labor, and z is an idiosyncratic productivity (manegerial ability) shock. This should capture the fact that entrepreneurial investments are risky. Each agent supplies his unit labor endowment inelastically and thus receives a riskless wage. The production function features the typical concavity assumptions $F_k > 0$, $F_l > 0$, $F_{kl} = F_{lk} > 0$, $F_{kk} < 0$ and $F_{ll} < 0$, where F_k (F_l) denotes the derivative of F with respect to its first (second) argument.

For now we assume that all agents are ex ante identical with respect to their ability, i.e. z is distributed iid across agents. Further, in this simple model we assume that z can only take two realizations: With probability p the project is successful and $z = \bar{z}$ realizes, with the complementary productivity $1 - p$ it fails and $z = 0$ realizes. To keep things simple, in this two period version of the model I further assume full depreciation of capital after production. The capital stock k needs to be installed in order for the entrepreneur to learn her productivity while labor input l is chosen after learning z.

In the following section we will characterize the laissez-faire equilibrium. We then will analyze a constrained planning problem, in which the planner is only allowed to choose the capital investment of each agent but restrained from performing any ex-post (i.e. after production takes place) transfers. It turns out that this already leads to a welfare improvement due to a pecuniary externality. Agents do not internalize the effect of their savings decision on wages while the planner does. Finally, we solve the Ramsey problem for a planner that has access to linear capital income taxes and lump-sum transfers.

2.1 The Laissez-Faire Economy

In this section we characterize the equilibrium in the laissez-faire economy. We will do so by backward induction. First, we will solve for the equilibrium in period two, given capital investments and realizations of the productivity shocks. This boils down to solving the labor market equilibrium as the only choice in the second period is the labor input that each firm demands.

Second Period. By assumption unsuccessful firms (with productivity shock $z = 0$) have zero output, irrespective of the labor input. These firms will hence not hire any workers. Let $k(a)$ be the capital investment of agents with initial wealth a and let w be the wage. The fraction p of those agents, which experiences the good shock $z = \bar{z}$, will hire labor in order to maximize firm profits

$$\pi(a) = \max_{l} \{F(\bar{z}k(a), l) - wl\}.$$

Differentiating with respect to labor yields the familiar first order condition for labor demand,

$$F_l(\bar{z}k(a), l) = w. \tag{1}$$

Under the imposed assumptions on F labor input is linear in $k(a)$. In particular optimal labor input can be decomposed as

$$l(w, a) = \tilde{l}(w)k(a),$$

where the coefficient $\tilde{l}(w)$ only depends on the wage. To see this, note that because of constant returns to scale the above problem is equivalent to solving for the optimal labor-capital ratio in

$$\max_{\frac{l}{k(a)}} F\left(\bar{z}, \frac{l}{k(a)}\right) - w\frac{l}{k(a)}.$$

Applying the implicit function theorem on the first order condition (1) one can further show that $l(a, w)$ is decreasing in the wage w.

Using this linearity property and the assumption that shocks are independently and identically distributed across agents we can write the labor market clearing condition as

$$p \int l(w,a)dG(a) = p\tilde{l}(w) \int k(a)dG(a) = pl(w)K = 1.$$

Hence, there is a one-to-one relationship between the equilibrium wage w and the aggregate capital stock K. In particular, $w = \tilde{l}^{-1}(\frac{1}{pK})$. This allows us to write labor input of an agent who started with assets a, invested an amount $k(a)$ in her firm and experienced the good shock, as

$$l(a) = \frac{1}{p}\frac{k(a)}{K}.$$

Profits of a successful entrepreneur with initial endowment a are hence given by

$$\pi(a) = \underbrace{\left[F\left(\bar{z}, \frac{1}{pK}\right) - \frac{w}{pK}\right]}_{r^k} k(a) = r^k k(a). \tag{2}$$

First Period. Using these results we are now able to characterize the optimal investment choices in the first period. The optimization problem of an agent with initial endowment a can be written as

$$\max_{k(a)} u(c_0(a)) + \beta\Big(pu(c_g(a)) + (1-p)u(c_b(a))\Big).$$

where consumption in the initial period is given by

$$c_0(a) = a - k(a). \tag{3}$$

Consumption in the second period depends on the realization of the shock. If the good state, $z = \bar{z}$, realizes it is

$$c_g(a) = r^k k(a) + w, \tag{4}$$

If the bad shock, $z = 0$, realizes consumption in the second period is

$$c_b(a) = w, \tag{5}$$

i.e. there are no firm profits and the agent simply consumes her wage income.

Moreover, capital investment needs to be non-negative,

$$k(a) \geq 0 \qquad \forall a. \tag{6}$$

The Euler equation for risky capital investment will be at the center of the discussion in the next section. In the laissez-faire market economy, it is given by

$$u'(c_0(a)) = \beta p u'(c_g(a)) r^k. \tag{7}$$

2.2 Constrained Efficiency

The laissez-faire equilibrium suffers from severe inefficiencies. This becomes most apparent when solving the problem of a constrained planner. To be specific, the concept we are using in this section is the one of *constrained efficiency*, which goes back to - at least - Diamond (1967). It does not allow the planner to overcome the frictions implied by missing markets. In particular the planner is not allowed to engage in any form of redistribution after the shocks are realized. Instead she is only allowed to make the investment choice on behalf of each agent. She thereby has to satisfy each agent's budget constraint.

We show that the planner's allocation obtained in this way features higher capital investment and is in general more efficient than the laissez-faire allocation. More specifically, if all agents are initially identical, the planner's allocation strictly Pareto dominates the laissez-faire allocation. If agents differ in their initial wealth holdings the improvement is in the utilitarian sense. In particular, while some (rich) agents are worse off in the planner's allocation compared to the laissez-faire, aggregate social welfare with equal Pareto weights on all agents is higher in the planner's allocation. The reason for these findings is that market incompleteness induces a "pecuniary externality". Agents do not internalize the effect of their savings choice on wages, while the planner does.

Closely related is the study by Davila, Hong, Krusell, and Rios-Rull (2012), who analyse this form of inefficiency in a set up very similar to mine, the only difference being that labor- instead of capital income is risky in their model. The authors find that the laissez-faire equilibrium is constrained inefficient and depending on the calibration there is over- or under-accumulation of capital compared to the constrained optimum. In contrast, in the present framework the laissez-faire allocation always features under-accumulation of capital. Also related is a recent paper by Gottardi, Kaji, and Nakajima (2016). The authors characterize constrained inefficiencies of the laissez faire equilibrium in a more general setting with both risky labor- and risky capital income. As in their paper the welfare effects of a change in equilibrium prices induced by changes in aggregate investment in the present paper can be decomposed into an insurance component and - if agents are ex-ante heterogeneous - a distribution component. The analysis of constrained efficiency in general and this decomposition in particular help to understand the optimal tax formulas in the Ramsey problem of the following section.

2.2.1 Ex Ante Identical Agents

Let us first investigate the case in which all agents have the same initial wealth A and only differ in their idiosyncratic returns to capital. In this case we can set $k(a) = K$.

The (constrained) planner's problem then reduces to

$$\max_K u(c_0) + \beta\Big(pu(c_g) + (1-p)u(c_b)\Big),$$

where the consumption levels are given by

$$c_0 = A - K, \tag{8}$$

$$c_g = r^k K + w, \tag{9}$$

and

$$c_b = w, \tag{10}$$

and equilibrium prices are determined by the capital stock the planner chooses,

$$w = F_L\left(\bar{z}, \frac{1}{pK}\right) \quad \text{and} \quad r^k = F\left(\bar{z}, \frac{1}{pK}\right) - \frac{w}{pK}.$$

The first order condition with respect to capital is given by

$$u'(c_0) = \beta p u'(c_g) r^k + \beta\left[p\left(\frac{\partial r^k}{\partial K}K + \frac{\partial w}{\partial K}\right) + (1-p)\frac{\partial w}{\partial K}\right]. \tag{11}$$

Using equation (2) we can rewrite the derivative of the return on capital with respect to capital as

$$\frac{\partial r^k}{\partial K} = \frac{1}{pK^2}\underbrace{\left[-F_l\left(\bar{z}, \frac{1}{pK}\right) + w\right]}_{=0} - \frac{1}{pK}\frac{\partial w}{\partial K}, \tag{12}$$

where we plugged in the first order condition for labor demand (1).

Thus we can rewrite the Euler equation (11) as

$$u'(c_0) = \beta p u'(c_g) r^k + \Delta, \tag{13}$$

where

$$\Delta = \beta \frac{\partial w}{\partial K}(1-p)\Big(u'(c_b) - u'(c_g)\Big) \geq 0 \tag{14}$$

is the additional term compared to the laissez-faire equilibrium. This term is strictly bigger than zero whenever $p < 1$. Moreover Δ is decreasing in p. Intuitively, a higher capital stock increases the marginal product of labor and hence wages but reduces the marginal product of capital and hence firm profits. As both the lucky and the unlucky agents receive wages for labor but only the lucky ones receive firm profits, this provides valuable insurance. The laissez-faire equilibrium hence features under-accumulation of capital compared to the constrained optimum.

2.2.2 Ex Ante Heterogeneous Agents

We now assume that agents differ with respect to their initial wealth holdings and the planner is choosing $k(a)$ for each initial wealth endowment a. The (constrained) utilitarian planner's problem is given by

$$\max_{\{k(a)\},K} \int \left(u(c_0(a)) + \beta \Big(pu(c_g(a)) + (1-p)u(c_b(a)) \Big) \right) dG(a),$$

where c_0, c_g and c_b are again given by the identities (3) to (5), the capital investment of each agent is again required to be non-negative, $k(a) \geq 0$ and prices are again given by (12). It is convenient to make the aggregate capital stock K a choice of the planner. Doing so requires to impose the constraint

$$K = \int k(a) dG(a). \tag{15}$$

The Euler equation for capital investment $k(a)$ is given by

$$u'(c_0(a)) = \beta p u'(c_g(a)) r^k + \mu, \tag{16}$$

where μ denotes the Lagrange multiplier for condition (15) and can be interpreted as the marginal second period social benefit of increasing K,

$$\mu = \beta \frac{\partial w}{\partial K} \int \left\{ (1-p)u'(c_b(a)) - \left(\frac{k(a)}{K} - p \right) u'(c_g(a)) \right\} dG(a). \tag{17}$$

We can decompose the integrand into an insurance and a redistribution component, i.e. μ can be written as

$$\mu = \beta \frac{\partial w}{\partial K} \int \Bigg\{ \underbrace{(1-p)\Big(u'(c_b(a) - u'(c_g(a)) \Big)}_{I(a)} + \underbrace{\left(1 - \frac{k(a)}{K} \right) u'(c_g(a))}_{D(a)} \Bigg\} dG(a) \geq 0. \tag{18}$$

It can be seen that this expression generalizes expression (14) to the case of heterogeneous initial wealth. If agents are ex-ante identical, $k(a) = K$ for all a and as a consequence $\mu = \Delta$ holds. The redistribution component $D(a)$ is zero in this case. With initial wealth heterogeneity the term $I(a)$ is positive for all a, while $D(a)$ is not. The sign of the redistribution component depends on the extent to which the agent is invested in her firm. It is positive exactly if the agent has less than average capital investment, $k(a) < K$.

Contrary to the case with ex-ante identical agents there is no unanimous support for the constrained efficient allocation. In particular, richer agents are in favour of the laissez-fare allocation. However behind the veil of ignorance, i.e. in a situation where agents do not know how many assets they are born with but just that they draw them from the distribution $G(a)$, there would be unanimous support. The reason is that the forces with which prices react to a change in savings compared to the laissez-faire economy foster ex-post redistribution which implies ex-ante insurance. To see this consider for simplicity the situation without production risk, i.e. $p = 1$, but uncertainty regarding the initial endowment. In this case μ is given only by the redistribution component

$$D = \beta \int D(a) dG(a) = \beta \frac{\partial w}{\partial K} \int \left(1 - \frac{k(a)}{K}\right) u'(c_g(a)) dG(a) > 0.$$

On average the term in round brackets is equal to zero. But since the marginal utility of consumption $c_g(a)$ is decreasing in a, while capital investment $k(a)$ is increasing in a the whole term is positive. Hence the utilitarian social planner would invest more in capital than agents in the laissez-faire allocation even when there is no idiosyncratic investment risk. The reason for this result is that poor agents derive most of their income from wages while rich agents derive most of their income from capital returns. An increase in the capital stock increase the former but decrease the latter. As - through a higher marginal utility - poor agents enter in the utilitarian social welfare function with a higher weight than rich agents, an increase in the capital stock leads to an improvement.

Note that in either case (with and without idiosyncratic investment risk), while there is a redistributive motive behind this result, it does not depend on transfers across agents. In the tax implementation below we show how this allocation can be achieved without cross-subsidization. The mechanism solely works via the impact of changes in K on prices. As due to the concavity of $u(.)$ consumption poor agents enter with a higher weight in utilitarian planner's objective and the share of labor income relative to total income of these agents is high, a welfare improvement can be achieved when wages are higher and returns on capital are lower.

It is worth comparing these results with those in Davilla et al (2012), who performed a similar analysis with idiosyncratic wage risk instead of investment risk. In this case two opposing forces determine the sign of the tax/subsidy on capital income. First, in the two period model with ex-ante identical agents a reduction of capital investment is optimal as it downscales risky wage income (through a lower marginal product of labor) and it upscales the riskless return on capital (through an increase in the marginal product of capital). Instead, with no risk but heterogeneous initial wealth the same analysis as here goes through. Since consumption poor agents derive their income mainly from wages, the utilitarian social planner would subsidize capital. In the multiple period version of their model with homogeneous initial assets, a dispersion of wealth endogenously emerges as a consequence of risky wage income and the two effects are counteracting each other. Which one dominates crucially depends on the calibration of the stochastic labor productivity process. Instead, in the present framework, both effects work in the same direction. Risky capital income as well as the fact that consumption poor agents derive most of their income from labor call for an increase in wages, which is achieved by a higher capital stock.

2.2.3 Implementation of the Constrained Optimum

We now want to think of how the constrained efficient allocation can be decentralized via appropriate tax incentives. A tax system that implements the constrained optimum must ensure that the individual's Euler equation is given by (16). In particular one can implement the constrained optimum via a subsidy $\tau(a)$ on capital investment financed by lump-sum - but wealth dependent - transfers. This subsidy is in general wealth dependent but transfers are chosen such that net government transfers to each single agent are equal to zero.

The consumer's Euler equation is then given by

$$u'(c_0(a)) = \beta \left\{ pu'(c_g(a)) \left(r^k + \tau(a) \right) + (1-p)u'(c_b(a))\tau(a) \right\}.$$

Denoting the inverse of the average marginal period two consumption of an agent with initial assets a by

$$\chi(a) = \frac{1}{pu'(c_g(a)) + (1-p)u'(c_b(a))} > 0,$$

the subsidy for this agent is given by

$$\tau(a) = \chi(a)\frac{\partial w}{\partial K} \int \left[(1-p)u'(c_b(\tilde{a})) - \left(\frac{k(\tilde{a})}{K} - p \right) u'(c_g(\tilde{a})) \right] dG(a). \qquad (19)$$

The lump sum tax $T(a) = \tau(a)k(a)$ then ensures that there are no net government transfers to any agent.

Let us inspect equation (19) a bit further. First consider the case of initial wealth equality. In this case the subsidy is the same for each agent and equals

$$\tau = \frac{\partial w}{\partial K}\left(1 - \frac{u'(c_g)}{pu'(c_g) + (1-p)u'(c_b)} \right) > 0.$$

It depends positively on the riskiness of the entrepreneurial project. Too see this note that the last term denotes the marginal utility of consumption in the good state divided by the average marginal utility of consumption in the second period. This fraction is smaller than one and decreasing in the variation of second period consumption, which is solely determined by the riskiness of entrepreneurial investment. The higher entrepreneurial risk the more severe is the pecuniary externality and hence the stronger the need for government intervention.

Now let initial wealth be heterogeneous across agents but assume that that $p = 1$, i.e. no investment risk. Then

$$\tau(a) = \chi(a)\frac{\partial w}{\partial K} \int \left[\left(1 - \frac{k(\tilde{a})}{K} \right) u'(c_g(\tilde{a})) \right] dG(\tilde{a}) > 0.$$

It is strictly positive as on average $k(\tilde{a})/K$ is equal to one, $k(a)$ is increasing and $u'(c_g(a))$ is decreasing in a.

Again we see that both model ingredients, risky investment and heterogeneity call for a positive subsidy. The reason is that the utilitarian welfare criterion imposes higher weight on consumption poor. As these agents derive most of their income from labor, an increase in the wage is welfare improving. Further note that $\chi(a)$ is increasing in a, implying that the subsidy $\tau(a)$ is increasing in initial wealth a. Note also that in contrast to Davila et al (2012), who implemented the constrained efficient allocation via a tax/subsidy on capital *income*, we subsidize capital *investment*. The reason is intuitive. While a subsidy on capital income also encourages investment it has less desirable insurance properties. In particular ex-post only lucky agents benefit from it.

2.3 The Ramsey Problem with Linear Capital Income Taxes

Constrained efficiency is a useful benchmark as it isolates the maximal welfare improvements that policy can achieve without any redistribution of wealth but only through its influence on equilibrium prices. In practice, however, certain policies are able to improve on the constrained efficient allocation by distributing from lucky to unlucky agents. This is the direction we want to explore in this section. To be specific, we want to allow the planner to choose a linear tax on capital income, the revenues of which are distributed lump sum to all agents.

As before, we first consider the case of ex-ante identical agents, after which we analyze the case of initial wealth heterogeneity.

2.3.1 Ex Ante Identical Agents

In the case of ex-ante identical agents the planning problem is given by

$$\max_{\tau} u(c_0) + \beta \Big[pu(c_g) + (1-p)u(c_b) \Big]$$

subject to the budget constraints

$$c_0 = A - K,$$

$$c_g = (1-\tau)r^k K + w + T,$$

and

$$c_b = w + T,$$

the government budget clearing condition

$$T = \tau p r^k K,$$

and the implementability constraint

$$u'(c_0) = \beta p(1-\tau)r^k u'(c_g). \tag{20}$$

The last constraint is simply the first order condition with respect to capital of the agent. Instead of directly choosing the allocation for each agent, the planner takes optimization behaviour of agents into account when optimizing over the tax rate τ. She further takes into account that wages and the return on capital in the good state are determined by the aggregate capital stock,

$$w = F_L\left(\bar{z}, \frac{1}{pK}\right) \quad \text{and} \quad r^k = F\left(\bar{z}, \frac{1}{pK}\right) - \frac{w}{pK}.$$

The solution to this problem can be summarized by a simple first order condition

$$u'(c_0)\frac{\partial c_0}{\partial \tau} + \beta\left[pu'(c_g)\frac{\partial c_g}{\partial \tau} + (1-p)u'(c_b)\frac{\partial c_b}{\partial \tau}\right] = 0. \tag{21}$$

We now interpret the partial derivatives entering this equation. Note that the only choice of the agent in the first period is capital investment k. This choice has an indirect effect on prices that the planner needs to take into account. The partial derivative of initial consumption with respect to τ is trivial,

$$\frac{\partial c_0}{\partial \tau} = -\frac{\partial K}{\partial \tau}.$$

It says that the increase in initial consumption due to a marginal increase in the tax rate is equal to the reduction in capital investment induced by this tax change. The partial derivative of consumption in the good state with respect to the tax rate is the most involved but we can decompose the change of consumption in the good shock induced by a change in the tax rate into effects that each belong to one of three categories

$$\frac{\partial c_g}{\partial \tau} = \underbrace{-r^k K + pr^k K}_{\text{Direct Tax Effects}} + \underbrace{\frac{\partial K}{\partial \tau} r^k(1-\tau+p\tau)}_{\text{Effects of Change in Savings}} + \underbrace{\frac{\partial w}{\partial K}\frac{\partial K}{\partial \tau}\left(1 - \frac{1-\tau}{p} - \tau\right)}_{\text{Effects of Change in Wages}}.$$

First, and most straight forward, a marginal increase in the tax rate reduces capital income and increases lump sum transfers. Absent any change in behaviour an increase in the tax rate by a marginal unit reduces capital income of the lucky agent by $r^k K$ units. At the same time it increases total revenue - which is rebated lump sum to all agents - by $pr^k K$ units.

Second, a change in the tax rate influences the investment behaviour of agents, which in turn affects consumption. In particular an increase in the tax rate will change capital investment by $\partial K/\partial \tau$ units. This (typically negative) derivative multiplied by $r^k(1-\tau)$ is the reduction of net capital income resulting from lower investment. Moreover, tax revenues - and hence lump sum transfers - will change by $(\partial K/\partial \tau)r^k p\tau$.

Finally, the change in savings behaviour influences wages. Complementarity of capital and labour implies that $\partial w/\partial\tau$ and $\partial K/\partial\tau$ have the same (typically negative) sign. If an increase in the tax rate leads to a reduction in investment, it also leads to a reduction in wages. The labor income of agents is hence reduced exactly by $\partial w/\partial\tau$. But the reduction in wages also increases the capital income of lucky entrepreneurs. As each agent who receives the good shock $\bar{z}$ - i.e. a share p of all agents - hires $1/p$ units of labor, the net capital income of those agents is increased by $(-\partial w/\partial\tau)(1-\tau)/p$ units. The last term $(-\partial w/\partial\tau)\tau$ denotes the taxed part of the increase in gross profits, which is rebated lump sum to all - i.e. also unlucky - agents.

The partial derivative of consumption in the bad state with respect to the tax rate comprises simply of a subset of the terms above,

$$\frac{\partial c_b}{\partial\tau} = pr^k K + \frac{\partial K}{\partial\tau} r^k p\tau + \frac{\partial w}{\partial K}\frac{\partial K}{\partial\tau}(1-\tau) = \frac{\partial w}{\partial K}\frac{\partial K}{\partial\tau} + \frac{\partial T}{\partial\tau}.$$

As explained above the labor income of the unlucky agent is the same as for the lucky agent. An increase in the tax rate changes this labor income by $\partial w/\partial\tau =$ units, which by the chain rule is given by $\partial w/\partial\tau = (\partial w/\partial K)(\partial K/\partial\tau)$. As unlucky agents neither receive capital income, nor pay any capital income taxes, all the other effects can be summarized in the change in the partial derivative of lump sum transfers with respect to τ,

$$\frac{\partial T}{\partial\tau} = pr^k K + \frac{\partial K}{\partial\tau} r^k p\tau - \frac{\partial w}{\partial K}\frac{\partial K}{\partial\tau}\tau. \tag{22}$$

This derivative can again be decomposed in a direct tax revenue effect $pr^k K$, the change in tax revenue through a reduction in savings $r^k p\tau\partial K/\partial\tau$ and the change in tax revenue through an increase in capital income induced by a decrease in wages $\tau\partial w/\partial\tau$.

Plugging in these derivatives as well as the implementability constraint (20) into the first order condition (21) and rearranging terms gives an intuitive formula for the optimal tax rate. For this means it is useful to define the elasticity of capital investment with respect to the net-of-tax rate as

$$\epsilon_{K,1-\tau}(\tau) = \frac{\partial K(\tau)}{\partial(1-\tau)}\frac{1-\tau}{K(\tau)} > 0 \tag{23}$$

and the elasticity of the equilibrium wage with respect to aggregate capital as

$$\epsilon_{K,1-\tau}(K) = \frac{\partial w(K)}{\partial K}\frac{K}{w(K)} > 0. \tag{24}$$

The optimal tax formula then reads

$$\frac{\tau}{1-\tau} = \frac{(1-p)[u'(c_b) - u'(c_g)]}{\mathbb{E}u'(c_1)}\left(\frac{1}{\epsilon_{K,1-\tau}(\tau)} - \epsilon_{w,K}(K)\frac{w}{pr^k K}\right) \tag{25}$$

Let us interpret this formula. The right hand side is the product of two terms. The first term denotes the difference between marginal utilities in the bad and the good state multiplied by the probability of failure $1 - p$ and normalized by the expected time one marginal utility. This term is unambiguously positive as, for capital investment to be positive, consumption in the good state needs to be higher than consumption in the bad state. The bigger the difference, the higher are insurance gains from distorting the investment decision.

The sign of the second term is ambiguous implying that in general both a capital income tax as well as a capital income subsidy can be optimal. The sign depends on the difference of two - interacting - terms, both of which are positive. The first term is the inverse of the elasticity of capital investment with respect to the net-of-tax rate. It describes the increase of capital investment after a marginal reduction in the tax rate. The second is the elasticity of the equilibrium wage with respect to the aggregate capital stock multiplied by the ratio of aggregate labor income and aggregate (gross) capital income. The higher any of these elasticities the lower the (potentially negative) tax rate. Absent any responses of agents an increase in the tax rate allows for ex-post redistribution of capital income, i.e. ex-ante insurance of second period consumption. However, an increasing tax rate in general reduces capital investment and hence the tax base (first term). Moreover, the resulting reduction in the aggregate capital stock reduces wages, which - as they are received by all households - also serve as insurance against the adverse shock. This second effect is the more important the higher the ratio of aggregate labor income and aggregate capital income.

A simple back of the envelope calculation already provides an indication that this second pecuniary effect might be quantitatively important. Consider the standard parameterization of the neoclassical growth production function, in which the ratio of labor- to capital income is around two and the elasticity of wages with respect to capital is 1/3, implying that the second term is 2/3. In a recent paper Devereux et al (2014) find that the elasticity of taxable income with respect to the corporate income tax rate is between 0.13 and 0.17 for companies with profits around 300,000 pounds and between 0.53 and 0.56 for companies with profits around 10,000 pounds. For simplicity let us take the somewhat medium value of 1/3. In my model this elasticity corresponds to

$$-\frac{\partial \ln pr^k K}{\partial \ln \tau} = \frac{1}{3}.$$

Again with the standard parameterization of the neoclassical production function one obtains $\partial \ln r^k / \partial \ln K = -2/3$, implying

$$-\frac{\partial \ln pr^k K}{\partial \ln \tau} = -\left(\frac{\partial \ln r^k}{\partial \ln K} + 1\right)\frac{\ln K}{\ln \tau} = -\frac{1}{3}\frac{\ln K}{\ln \tau} = \frac{1}{3}.$$

Hence our estimate for the elasticity is

$$\epsilon_{K,1-\tau} = -\frac{\ln K}{\ln \tau} = 1.$$

Therefore,

$$\frac{1}{\epsilon_{K,1-\tau}(\tau)} - \epsilon_{w,K}(K)\frac{w}{pr^k K} \approx 1 - \frac{2}{3}$$

This implies that general equilibrium effects on wages influence the optimal tax rate substantially. If, for example, the optimal tax rate in partial equilibrium (without responses in wages) is 50%, it would be reduced by half to 25% when accounting for the reaction on wages. Moreover, the general equilibrium effect becomes relatively larger, the smaller the partial equilibrium tax. If, for example, the optimal partial equilibrium tax rate is only 25%, it would be reduced by more than two thirds to below 8%.

2.3.2 Ex Ante Heterogenous Agents

Let us now consider the case of initial wealth heterogeneity. In this case the planning problem is given by

$$\max_{\tau} \int \left\{ u(c_0(a)) + \beta \left[pu(c_g(a)) + (1-p)u(c_b(a)) \right] \right\}$$

subject to the budget constraints

$$c_0(a) = a - k(a),$$

$$c_g(a) = (1-\tau)r^k k(a) + w + T,$$

and

$$c_b(a) = w + T,$$

the government budget clearing condition

$$T = \tau p r^k K,$$

and the implementability constraint

$$u'(c_0(a)) = \beta p(1-\tau)r^k u'(c_g(a)).$$

Wage and capital returns are again given by

$$w = F_L\left(\bar{z}, \frac{1}{pK}\right) \quad \text{and} \quad r^k = F\left(\bar{z}, \frac{1}{pK}\right) - \frac{w}{pK},$$

where the aggregate capital stock is

$$K = \int k(a) dG(a).$$

The solution to this problem can again be summarized by a simple first order condition

$$\int \left\{ u'(c_0(a))\frac{\partial c_0}{\partial \tau} + \beta\left[pu'(c_g(a))\frac{\partial c_g(a)}{\partial \tau} + (1-p)u'(c_b(a))\frac{\partial c_b(a)}{\partial \tau}\right]\right\} = 0. \qquad (26)$$

The partial derivatives of the respective consumption levels with respect to the tax rate are given as follows. The derivative of initial consumption with respect to the tax rate is analogous to the case with ex-ante identical agents,

$$\frac{\partial c_0(a)}{\partial \tau} = -\frac{\partial k(a)}{\partial \tau}.$$

The other two derivatives are slightly more involved. The difference to the case of identical initial wealth comes from the fact that the aggregate capital stock now does not equal the individual capital investments. The derivative of consumption in the good state with respect to the tax rate is given by

$$\frac{\partial c_g(a)}{\partial \tau} = \underbrace{-r^k k(a) + pr^k K}_{\text{Direct Tax Effects}} + \underbrace{r^k \left[\frac{\partial k(a)}{\partial \tau}(1-\tau) + \frac{\partial K}{\partial \tau}p\tau\right]}_{\text{Effects of Change in Savings}} + \underbrace{\frac{\partial w}{\partial K}\frac{\partial K}{\partial \tau}\left(1 - \frac{1-\tau}{p} - \tau\right)}_{\text{Effects of Change in Wages}}.$$

We can again decompose the derivative into three sets of effects: direct tax effects, effects through changes in savings, and effects through a change in wages. The interpretation of these effects is the same as above. Note that the effects of changes in savings include two different partial derivatives. The first is the derivative of individual saving with respect to the tax rate. It captures the fact that a change in taxes changes the (net) return on capital and therefore induces changes in investment. The second is the derivative of the aggregate capital stock with respect to the tax rate. It captures the reduction in lump sum transfers that a reduction in aggregate capital investment induces. The distinction arises because other's savings behaviour does not affect individual profits (ignoring the reaction on wages captured in the last term). It affects, however, the total tax revenue and hence lump sum transfers to the individual.

The derivative of consumption in the bad state with respect to the tax rate is almost the same as above.

$$\frac{\partial c_b(a)}{\partial \tau} = pr^k K + \frac{\partial K}{\partial \tau}r^k p\tau + \frac{\partial w}{\partial K}\frac{\partial K}{\partial \tau}(1-\tau) = \frac{\partial w}{\partial K}\frac{\partial K}{\partial \tau} + \frac{\partial T}{\partial \tau}.$$

The reason is that individual consumption in the bad state is affected only by changes in aggregate investment as the return on individual investment is zero no matter what the tax rate is. Similarly to above we can decompose the partial derivative into an effect on wages and an effect on transfers, where the latter is given by

$$\frac{\partial T}{\partial \tau} = pr^k K + \frac{\partial K}{\partial \tau}r^k p\tau - \frac{\partial w}{\partial K}\frac{\partial K}{\partial \tau}\tau.$$

Plugging these partial derivatives into the first order condition (26), substituting out $u'(c_0(a))$ using the implementability condition and rearranging terms again yields an intuitive formula for the optimal tax rate,

$$\frac{\tau}{1-\tau} = \frac{\int \left\{ (1-p)\left[u'(c_b(a)) - u'(c_g(a))\right] + (1 - \frac{k(a)}{K})u'(c_g(a))dG(a) \right\}}{\mathbb{E}u'(c_1)} \times \left(\frac{1}{\epsilon_{K,1-\tau}(\tau)} - \epsilon_{w,K}(K)\frac{w}{pr^k K} \right). \tag{27}$$

Let us compare this formula with the one for the case of identical wealth, equation (25). It is again the product of two terms. The second term is identical to before. The numerator of the first term now features an integral over initial wealth. The first term in this integral has the same interpretation as before. It describes the insurance motive of the optimal tax. The second term is new and describes the redistribution motive. As discussed in the section on constrained efficiency this term - as the first term - is unambiguously positive as $(1 - k(a)/K)$ is zero on average but the weights $u'(c_g(a))$ are decreasing in a.

It is important to note that this new redistributive motive affects the magnitude but not the sign of the optimal tax (subsidy). As before the sign is solely determined by the difference between the inverse of the elasticity of the capital stock with respect to the net-of-tax rate and the elasticity of the equilibrium wage with respect to the capital stock times the ratio of labor and capital income in the economy. Thus, in this framework, and contrary to conventional wisdom, an increase in the inequality of wealth may not necessarily imply a higher optimal tax on capital income. In fact, if the second term is negative, it would imply a higher optimal subsidization of capital. The reason for this result comes again from the pecuniary externality on wages. If both of the elasticities that enter the tax formula are high and hence capital investment and wages react strongly to changes in taxes, it is precisely a redistributive motive that makes Ramsey planner want to subsidize capital even more than with lower initial wealth inequality. Such a subsidization increases the equilibrium wage while decreasing the (gross) return on capital, benefiting poor agents, who derive the lion's share of their total income from wages.

3 The Infinite Horizon Economy

In progress.

4 Conclusion

To be written.

A Economy with Financial Intermediation

So far we assumed that firms do not obtain any external funds and that each agent can save only by means of (risky) investment in her firm. We will now deviate from these assumptions. In particular, we assume that there exists a perfectly competitive financial intermediation sector that collects deposits (riskless bond holdings b) and lends them to firms. Perfect competition implies that these loans price the enterprise's default risk actuarially fair. We exogenously assume that only conventional debt contracts are available as external financing form. The amount of external finance, debt $d(a)$, that can be raised by a firm is tied to the firm's capital stock. In particular, for now we assume that

$$d(a) \leq \xi k(a) \quad \forall a. \tag{28}$$

In reality there are several reasons, why external financing is constrained. These reasons include informational asymmetries, limited contract enforceability, moral hazard, etc. An extensive literature on financial contracting exists that combines subsets of these reasons and derives the optimal contract in the respective setting.[4] In fact, in many cases a conventional debt contract turns out to be optimal. It is beyond the scope of this paper to derive an optimal contract between the entrepreneur and the bank in the presence of a certain friction. Instead we assume the most conventional financing structure of enterprises from the outset. Moreover, instead of explicitly modelling such frictions we capture them in reduced form via the parameter ξ. The basic idea is that the entrepreneur needs to have sufficient "skin in the game" such that these frictions are overcome.

We further assume that when a firm's profits are not enough to pay back debt, the bank cannot seize an entrepreneur's private assets. The entrepreneur has limited liability and is liable only to the extent that the capital stock she invested in her firm is lost. This corresponds to the real life situation of incorporated businesses, where the entrepreneur and her firm are two separate legal entities. The debt contract is one between the firm and the bank, not between the entrepreneur and the bank.

This section is again divided into three subsections. We will first derive the laissez-faire equilibrium, where we will mainly focus on the differences compared to the equilibrium without a financial sector. We then will derive the constrained optimum and its tax implementation.

A.1 The Laissez-Faire Economy

The labor market equilibrium is characterized by the same conditions as in the economy without financial intermediation. It shall hence be omitted here. The main differ-

[4] Some of the more prominent work include, for example, Townsend (1979), Diamond (1984), Diamond and Hellwig (1985), Innes (1993), Hellwig (2000), etc.

ence to above is that there are two additional markets, one for riskless deposits b and one for risky entrepreneurial loans d. These markets need to clear in equilibrium.

Bond Market Equilibrium. Let q be the price at time $t = 0$ of a riskless bond that pays one unit of the endowment good in period $t = 1$ and let $\tilde{q}$ be the price of the entrepreneurial loan, which pays one unit of the endowment good next period if the project is successful and zero otherwise. In the latter case the firm defaults on its loan. Perfect competition in the financial sector then implies

$$\tilde{q} = pq. \tag{29}$$

Proposition 1. *Whenever $p < 1$ the constraint* (28) *holds with strict equality,*

$$d(a) = \xi k(a) \quad \forall a.$$

Proof. Let a be the endowment of the agent. Choose any feasible pair of capital investment and initial consumption, $k = \bar{k}$ and $c_0 = \bar{c}_0 = a - \bar{k} + q(pd - b)$. Then the expected return on the agent's portfolio

$$p\left(r^k \bar{k} - d\right) + b = pr^k k - pd + b$$

is the same for any feasible pair (b, d) compatible with $(\bar{k}, \bar{c}_0)$. Its variance, however, is minimized with $d = \xi k$. This choice is hence the optimum for risk-averse agents. Since this holds for any feasible pair $(\bar{k}, \bar{c}_0)$ it has to hold also for the optimal one. Hence all firms feature maximal leverage. □

The intuition is that it is efficient to shift as much risk as possible to risk-neutral lenders.[5] An immediate corollary of this proposition is that the aggregate demand for risky entrepreneurial bonds equals the fraction ξ of the aggregate capital stock, i.e. market clearing for risky firm debt is given by

$$\int d(a) dG(a) = \xi \int k(a) dG(a).$$

Banks in turn finance these loans via riskless bonds (deposits) issued to households. As only a fraction p of corporate loans are paid back the market clearing condition for riskless bonds is

$$\int b(a) dG(a) = p \int d(a) dG(a).$$

For notational convenience in what follows we will substitute out $d(a) = \xi k(a)$ and combine the former two market clearing conditions to

$$\int b(a) dG(a) = p\xi \int k(a) dG(a). \tag{30}$$

[5] Or equivalently, to lenders that have a perfectly diversified loan portfolio.

Individual Optimization. We now derive the agents' first order condition. An agent with initial endowment a solves

$$\max_{k(a),b(a)} u(c_0(a)) + \beta\Big(pu(c_g(a)) + (1-p)u(c_b(a))\Big).$$

subject to the initial budget constraint

$$c_0(a) = a - (1 - \tilde{q}\xi)k(a) - qb(a), \tag{31}$$

the period one budget constraints

$$c_g(a) = (r^k - \xi)k(a) + b(a) + w, \tag{32}$$

and

$$c_b(a) = b(a) + w, \tag{33}$$

as well as the financial constraints

$$k(a) \geq 0 \quad \text{and} \quad b(a) \geq -w \ \ \forall a. \tag{34}$$

The last inequality imposes the natural borrowing limit on private debt. Agents can hold a negative amount of bonds. They however need to be able to repay it under ever contingency, i.e. also when the bad shock realizes. Debt can hence not exceed the wage income.

The Euler equation for risky capital investment is now given by

$$(1 - \tilde{q}\xi)u'(c_0(a)) = \beta p u'(c_g(a))(r^k - \xi).$$

In addition an Euler equation for bonds need to hold,

$$qu'(c_0(a)) = \beta\bigg(pu'(c_g(a)) + (1-p)u'(c_b(a))\bigg).$$

Combining these two conditions by substituting out q we obtain

$$u'(c_0(a)) = \beta\bigg\{pr^k u'(c_g(a)) + (1-p)p\xi\bigg(u'(c_b(a)) - u'(c_g(a))\bigg)\bigg\}. \tag{35}$$

The right hand side of this condition is now different than in the Euler equation without financial intermediation, equation (7) above. In particular, there is an additional term, the second term in the curly brackets. Let us interpret this new condition. As before this equation balances the individual costs (left hand side) and benefits (right hand side) of reducing initial consumption c_0 by a marginal unit and increasing capital investment k by a marginal unit. But since now the entrepreneur only needs to raise the fraction $(1 - pq\xi)$ of this unit of capital herself, she can buy riskless bonds with the remaining fraction $pq\xi$. In particular she can buy $p\xi$ units of the riskless bond at price q. The second term in curly brackets describes the additional insurance obtained in this way. In the good state the entrepreneur's firm profits are reduced by ξ units as this is the debt repayment of an additional unit of capital. But in every case she receives $p\xi$ units due to higher savings in the riskless bond. Thus with probability p the entrepreneur looses $(1-p)\xi$ units and with probability $(1-p)$ she gains $p\xi$ units of second period consumption. As the latter is valued higher than the former - due to higher marginal utility in the bad state - the whole term is positive.

A.2 Constrained Efficiency

Let us now investigate the constrained efficient allocation for the case with financial intermediation. As before we restrict the planner to take the market structure as given. In particular, the only source of external funds the planner can allocate to a firm are conventional debt contracts. Similarly we do not allow her to overcome the exogenously imposed financial friction, i.e. the amount of debt is limited in the same way as in the laissez-faire case. As before we first analyze the case of no initial wealth heterogeneity first.

A.2.1 Ex Ante Identical Agents

The planner solves the problem

$$\max_K u(c_0) + \beta\Big(pu(c_g) + (1-p)u(c_b)\Big)$$

subject to the budget constraints

$$c_0 = A - K,$$

$$c_g = K(r^k - \xi(1-p)) + w,$$

and

$$c_b = p\xi K + w.$$

The first order condition for capital investment is given by

$$u'(c_0) = \beta\Big\{pr^k u'(c_g) + (1-p)p\xi\Big(u'(c_b) - u'(c_g)\Big)\Big\} + \mu, \tag{36}$$

where μ is again the marginal social benefit of increasing the aggregate capital stock,

$$\mu = \beta(1-p)\frac{\partial w}{\partial K}\Big(u'(c_b) - u'(c_g)\Big),$$

and takes the same form as without external finance.

Compared to the case without financial intermediation, a higher capital stock not only has desirable insurance properties through its effect on wages. It also increases the supply of riskless bonds in the economy. In particular every additional unit of capital increases the supply of riskless bonds by $p\xi$ units. Similar to income from wages, also income from bonds is the same in both states and hence serves as insurance. The difference is that agents internalize the latter but not the former.

A.2.2 Ex Ante Heterogeneous Agents

With initial wealth heterogeneity the planner solves

$$\max_{\{k(a),b(a)\},K} \int \left(u(c_0(a)) + \beta\Big(pu(c_g(a)) + (1-p)u(c_b(a)) \Big) \right) dG(a),$$

subject to the initial budget constraint

$$c_0(a) = a - (1 - \tilde{q}\xi)k(a) - qb(a), \tag{37}$$

the period one budget constraints

$$c_g(a) = (r^k - \xi)k(a) + b(a) + w \tag{38}$$

and

$$c_b(a) = b(a) + w, \tag{39}$$

the financial constraints

$$k(a) \geq 0 \quad \text{and} \quad b(a) \geq -w \;\; \forall a, \tag{40}$$

the identity

$$K = \int k(a) dG(a),$$

as well as the bond market clearing condition

$$\int b(a) dG(a) = p\xi \int k(a) dG(a). \tag{41}$$

Denoting the Lagrange multiplier on the bond market clearing condition (41) by ν, the first order condition with respect to $k(a)$ is given by

$$(1 - \tilde{q}\xi)u'(c_0(a)) = \beta p u'(c_g(a))(r^k - \xi) + \mu + p\xi\nu$$

and the first order condition with respect to $b(a)$ by

$$qu'(c_0(a)) = \beta\Big(pu'(c_g(a)) + (1-p)u'(c_b(a)) \Big) - \nu.$$

Combining these equations by substituting out ν and rearranging terms yields

$$u'(c_0(a)) = \beta\Big\{ pr^k u'(c_g(a)) + (1-p)p\xi\Big(u'(c_b(a)) - u'(c_g(a)) \Big) \Big\} + \mu, \tag{42}$$

the analogue to the homogeneous case. Also in the heterogeneous case the Lagrange multiplier μ takes the same form as without financial intermediation,

$$\mu = \beta \frac{\partial w}{\partial K} \int \Bigg\{ \underbrace{(1-p)\Big(u'(c_b(a) - u'(c_g(a)) \Big)}_{I(a)} + \underbrace{\left(1 - \frac{k(a)}{K} \right) u'(c_g(a))}_{D(a)} \Bigg\} dG(a) \geq 0.$$

B Corporate Sector (in progress)

While entrepreneurship is an important component of any developed economy, there are also big corporations, not only small privately held businesses. We now want to investigate the implications on constrained efficiency of including such a corporate sector. In particular assume that there is a representative firm, which produces output according to the same production function

$$Y^c = F(K^c, L^c)$$

as entrepreneurs, the only difference being that capital augmenting productivity is deterministic. For simplicity we normalize it to one, $z^c = 1$. We can decompose the aggregate production inputs in a component that is used in the entrepreneurial sector and one that is used in the corporate sector,

$$K = K^e + K^c \quad \text{and} \quad L = L^e + L^c = 1.$$

Depending on the parameterization there are three possible equilibria, one in which all production takes place in the corporate sector $K^c = K$, one in which all production takes place in the entrepreneurial sector $K^e > K$, and an intermediate case with both $K^c > 0$ and $K^e > 0$. We want to restrict attention to the latter case. A necessary and sufficient assumption for this is $p\bar{z} > 1$. Under this assumption average productivity in the entrepreneurial sector is higher than in the corporate sector. Hence the unconstrained optimum is identical to the one without a corporate sector, i.e. $K^c = 0$.

B.0.1 Laissez-Faire Equilibrium

In order to characterize the market equilibrium we again focus on the main differences to our benchmark model without external finance. If $K^c > 0$ equilibrium wages and interest rates are determined by the marginal products in the corporate sector. In particular

$$w = F_l(K^c, L^c) \quad \text{and} \quad R = F_k(K^c, L^c). \tag{43}$$

Further bond market clearing is now given by

$$\int b(a) dG(a) = K^c,$$

as the capital stock of the corporate sector is financed by bond savings of households.

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