Sharing Risk to Avoid Tragedy: Theory and Application to Farming

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Abstract

Motivated by co-operational patterns of traditional rural societies, I present a model of joint interaction between risk sharing and common good co-operation in presence of voluntary participation constraints. Firstly, I show that not only risk sharing facilitates eliciting co-operation, but also the latter potentially improves the former, leading to Pareto improvements. Consequently, I analyze the complementarity between the two institutions. Lastly, I show three ways to decentralize the allocations using Arrow-Debreu securities with borrowing limits and action-dependent interest rates, tax or cap-and-trade mechanisms. To apply the model, I show indicative evidence that rural villages in India may be actually benefiting from such interactions. Finally, I calibrate a quantitative model to understand the productivity and insurance gains thereof.

Key words: Risk Sharing, Limited Commitment, Common Goods, Voluntary Participation, Village Economies, Macro-Development

JEL Codes: E20, Q54, D86

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1 Introduction

Production in many traditional societies often relies (or relied) on some kind of common goods that can be potentially free-ridden upon.[1] For instance, Ostrom (1990) provides an in-depth field study of societies that managed to co-operate over their common resources for very long periods of time, such as e.g. [1] the villagers living in Swiss mountains and Japan sharing grazing meadows, forests, "waste" lands, irrigation systems and roads; and [2] the societies living around Valencia, Murcia-Orihuela and Alicante in Spain and around the Ilocanos area in Philippines co-operating over the use of water canals for irrigating fields.

What is more, such traditional societies are (or were) living in the underdeveloped parts of the world characterized by high risk environments, e.g. due to the risks of natural disasters, variable crop yield, hunting results, pests and plant or human diseases. Such circumstances do constitute a rationale for the development of risk sharing mechanisms, be it in form of transfers of goods between the family or society members, diversification of farmers' landholdings or establishing informal credit markets. Indeed, the societies studied by Ostrom (1990) were usually not only co-operating over the common resources but also engaging in various forms of risk sharing. For instance, the Swiss villagers were insuring each other by rebuilding together houses destroyed by avalanches and by dividing the village's cheese production. In the case of Spanish societies, the contract on the water use also contained water sharing rules in times of draughts. In particular, the crops in most need of water were given priority. In case of the Filipino farmers, the way of dividing land was symmetrical in the sense of everyone having some land closer to and further away from the water source; and during dry periods all the farmers were collectively deciding how to share the burden and assign the water rights, again with priority for the crop in most need.

The aim of this paper to theoretically investigate the interaction between the two aforementioned institutions of risk sharing and co-operation over the common goods, and ultimately to quantitatively measure the insurance and productivity gains (or losses) associated with it. In order to do so, I first highlight the mechanism in a simple static model of risk sharing and common good co-operation. The main intuition is that risk sharing provides a punishment for deviations on the common good promises (in form of exclusion from future insurance) and so helps elicit the co-operation. Moreover, optimizing over the common goods may improve the extent of risk sharing. As a consequence, [1] agents facing higher idiosyncratic risk are more likely to be able to solve the common good problem; and [2] the ones who manage to do so, may benefit from better consumption smoothing.

Then, I use the model to derive empirically testable hypotheses which I validate with an ICRISAT panel dataset from Indian villages. To do so I draw upon the so called "rice theory"

[1] Dolsak and Ostrom (2003) define common goods (or, as they call it, "common-pool resources") as the ones with two characteristics: [1] rivalry (usage by one person subtracts from the ability of others to do the same); and [2] existence of a cost of excluding potential beneficiaries from access to the resource.

presented in Talhelm et al. (2014) stating that rice farming (as opposed to majority of other agricultural activities) necessitates a much larger degree of co-operation due to the more critical nature of the underlying common goods. By comparing rice farming households against the rest of the village, I find evidence in support of the mechanism outlined, namely that rice farmers [1] have relatively higher variance of idiosyncratic risk; and [2] have significantly better consumption smoothing.

Given this evidence, I extend the canonical model of risk sharing with limited commitment developed by Kehoe and Levine (1993) and Kocherlakota (1996) by the feature of common good affecting the production process and use it to quantitatively understand the extent of possible insurance and productivity gains of rural farmers in developing countries where such interaction between the two institutions is present. [IN PROGRESS]

The model outlined can be applied to variety of economic problems. For instance, it could be used to understand the tensions in fiscal unions with risk sharing mechanisms (such as e.g. common deposit insurance or unemployment insurance) and underlying common goods (such as the free trade market, common financial system structure, regional security or environment). Another prominent application might be to guiding our understanding of how to design more robust and comprehensive international environmental agreements.

Literature review [in progress]

My paper brings together two strands of the literature on risk-sharing with limited commitment and economics of natural resources.

This paper is also related to Abraham and Laczo (2016) where they introduce public and private storage. Similarly as the co-operation on usage of the common good analyzed in this paper, public storage relaxes participation constraints and improves risk sharing.

Furthermore, Abraham, Carceles-Poveda, Liu and Marimon (2016) show how to optimally design public risk sharing institution for the EMU. Based on the work of Alvarez and Jermann (2000), the Appendix A shows two possible ways of decentralizing the constrained efficient allocation using the Arrow-Debreu securities and: [1] the pollution permits (in the spirit of Lindahl (1954)); [2] the Pigouvian tax-subsidy schedule. Thus, I show that in presence of the common good externalities, the optimal design of such a public risk sharing institution should be combined with an instrument combatting the externality-related effects, such as the EU ETS. On the other hand, with the Pigouvian decentralization, my paper is second in the literature after Park (2016) to introduce endogenous taxation motives in models of risk sharing with limited commitment.

On the other hand, Genicot and Ray (2003) considered group deviations and showed that allowing for those implies an upper bound on the coalition size above which the risk sharing contract has no co-operative solution. While maintaining the assumption of unilateral deviations, I show that increasing the number of agents co-operating on the common good and risk sharing may still pose a threat to the agreement's stability.

Motivated by the Ostrom's (1990) field studies mentioned above, I apply rigorous methods to identify a possible underlying reason for why these traditional societies might have been able to sustain the effective long-term co-operation. In particular, I argue that risk sharing and solving the externalities related to the common goods with the institutions excluding non-cooperators, may be actually critical for the institutional reinforcement and for ensuring their long-term stability.

Similarly to the approach followed by me, the literature on international environmental agreements (IEAs; see survey by Barrett (2005)), has focused on self-enforcing contracts. As Kolstad and Toman (2001) point out, such agreements are likely to arise when there are only few parties responsible for the common good or the autarky threat is extremely high. These conditions do not fit well the case of environment which is [1] non-excludable in its nature; [2] has a large number of users; and [3] the threat of no cooperation is not that high. To this extent, the literature has also proposed various solutions facilitating voluntary participation - such as linking participation with trade agreements, financial transfers from rich to poor or punishments upon deviations. In this paper, I show that in presence of idiosyncratic shocks to productivity, the participation in IEAs may be facilitated with the use of appropriate risk sharing institutions.

Relatedly, due to the voluntary participation, countries may prefer short-term agreements in order to avoid the uncertainty associated with long-term commitments. In particular, Hardstad (2016) showed that due to hold up problems in negotiation stages and their impact on the new technology development decisions, short-term environmental agreements on emission levels can in fact be worse than having no agreement at all. Therefore, in order to combat the environmental pollution successfully, we should aim at establishing long-term contracts. Similarly, Battaglini and Harstad (2016) argued that participation becomes attractive only because large coalitions commit to long-term agreements that circumvent the associated hold-up problems. As the contract analyzed by me in this paper is long-term in its nature, I propose a mechanism that should facilitate this important aim. At the same time, however, the mechanism reveals potential threats to stability (or emergence) of such grand-coalitions.

This paper is organized as follows. In Section 2, I present the main mechanism in a static framework; Section 3 discusses the allocations with various degrees of co-operation over risk sharing and/or common good co-operation and the ways of decentralizing these [in progress]. Section 4 discusses possible complementarity and substitution between the two institutions. The empirical evidence motivating the application is in Section 5. Based on this evidence, Section 6 develops a quantitative model used to understand insurance and productivity gains due to the interaction of two institutions [in progress]. Finally, Section 7 concludes with a policy discussion [in progress].

2 Static Economy

Consider a one period economy with 2 risk averse agents that have rational expectations. Both of them are ex-ante identical and become ex-post heterogenous due to idiosyncratic productivity shocks. While producing, they make use of the common good that [1] is one of the factors determining the level of output; and also [2] is affected by their production decisions. The stochastic productivity and the underlying common good constitute possible motives for co-operation aimed at risk sharing (i.e. consumption smoothing) and/or [2] internalizing the externalities associated with agents' production.

More precisely, agents have GHH[2] preferences over consumption c and effort/labor supply n represented by $U(c,n) = \log\left(c - n^{\phi}\right)$ with[3] $n \in [0,1]$. Moreover, they produce using linear technology $y_i^0(\theta_i, n_i) = \theta_i n_i$ with $\theta_i \in \Theta = \left(\theta^1, ..., \theta^{N_\theta}\right)$, $0 < \theta^1 < ... < \theta^{N_\theta} < \infty$ being an i.i.d. productivity shock process with pdf $\pi(\theta)$ and the two first moments of $\mu = \mathrm{E}(\theta)$ and $\sigma^2 = \mathrm{Var}(\theta)$. For notational convenience, let $\theta_t \equiv \{\theta_1, \theta_2\}$ be the state variable of productivity realizations to both agents. Also, both the shock realizations and decisions on production are perfectly observable. In order to keep the analysis simple, I assume that agents cannot save nor borrow. [4]

In particular, the effective output equals $y_i = (1 - D(P(\theta)))\, y_i^0$ where $P(n_1(\theta), n_2(\theta)) = P(n_1(\theta)) + P(n_2(\theta))$ is the aggregate pollution level of the common good and $D : [0,1] \to [0,1]$ is the damage function measuring damages as a percent of the final-good output. As already mentioned, each agent's pollution depends on the chosen level of their labor supply n_i through the function $P(n_i(\theta_i))$ where $P : [0,1] \to [0,1]$ is an increasing and convex pollution function of n_i. Thus, it follows that $\frac{\mathrm{d}(1-D(P))}{\mathrm{d}n_i} = -D'(P)\frac{\partial P}{\partial n_i} < 0$ by all the previously made assumptions. Moreover, in order to ensure convexity of the production set, I assume:

Assumption 2.1: The common good damage function is concave, i.e. $\frac{\mathrm{d}^2(1-D(P))}{\mathrm{d}^2 n_i} \leq 0$.

For completeness of exposition, note that e.g. Dolsak and Ostrom (2003) define common goods (or, as they call it, "common-pool resources") as the goods with the following two characteristics: [1] rivalry (usage by one person subtracts from the ability of others to do the same); and [2] existence of a cost of excluding potential beneficiaries from access to the resource. While the first characteristic is clearly satisfied by the the above way of modeling common good, I will assume for now that the common good is fully excludable at no cost. In the Section XXX I discuss the impact of excludability on results derived.

The way of modeling damages to the output function in a multiplicative fashion pursued in this

[2]By abstracting from associated wealth effects, these preferences allow me to present the main results in a very simple environment. The dynamic model in Section XXX considers a more general class of utility functions.

[3]In what follows, I assume for simplicity that parameter ϕ is such that the constraint on $n$is never binding.

[4]If savings were observable, the qualitative nature of the main results would not change. See e.g. Abraham and Laczo (2017) for the impact of savings' unobservability on the properties of efficient risk sharing.

paper is similar to what Nordhaus and Boyer (2000) were proposing[5] in their integrated assessment models for one particular type of a common good: environment and the associated global warming.

Finally, co-operation over common goods may be particularly important in cases when the pollution accumulates (and depreciates) over time. The dynamic model developed in Section XXX presents an application allowing for this extension of the model.

3 Allocations with various degrees of co-operation

In this section, I first characterize the autarkic allocation with the tragedy of the commons and compare it to the first best benchmark. Then, I introduce voluntary and limited participation constraints and characterize the allocation where agents can voluntarily co-operate over solving the common good problem. Similarly, I outline the allocation with risk sharing only. Finally, I present the constrained efficient allocation of joint co-operation over the commons and risk sharing.

3.1 Non-cooperative allocation

Given the description of the model, the value of a [1] Nash equilibrium to agent i (when neither of the agents internalizes their externalities); or [2] equilibrium associated with a unilateral deviation from co-operation (if agent $-i$ adjusts his labor supply to account for externalities on others), conditional on productivity shock realization θ, is given by solution to the following problem:

$$V_i^{nc}\left(\theta,\gamma_i\left(\theta\right),n_{-i}\right) = \max_{\{\mathrm{c_i}(\theta),\mathrm{n_i}(\theta)\}}\log\left(\gamma_i\left(\theta\right)\left[c_i\left(\theta\right)-n_i^{\phi}\left(\theta\right)\right]\right) \quad (1)$$

$$s.t.$$

$$\left(\zeta_i\left(\theta\right)\right) \quad c_i\left(\theta\right) \leq \left(1-D\left(P\left(\theta\right)\right)\right)\theta_i n_i\left(\theta\right) \quad (2)$$

with n_{-i} and vector γ_i being given. For the time being, assume that $\gamma_i\left(\theta\right)=1\ \forall\theta$. Its role will become clear in Section 3.3 below.

After plugging in for consumption, the associated optimality condition with respect to labor supply n reads:

$$\left[\ \left(1-D\left(P\left(\theta\right)\right)\right)\theta_i+\tfrac{\mathrm{d}\left(1-D\left(P\left(\theta\right)\right)\right)}{\mathrm{d}n_i\left(\theta\right)}\theta_i n_i\left(\theta\right)\ \right] = \phi n_i\left(\theta\right)^{\phi-1} \quad (3)$$

First of all, the objective function is concave and the constraint set is convex. Thus, the FOC's are both necessary and sufficient. Sufficiency comes from the Assumption 2.1 guaranteeing concavity of the production function meaning that the associated production set exhibits non-increasing returns to scale.

[5]See also a more recent application of it to optimal carbon taxation by Golosov et al. (2014).

Secondly, the common good feature alters the standard trade-off associated with choosing optimal labor supply. In particular, apart from balancing marginal disutility of labor supply with the marginal benefit of increase in output, the agent takes also into account the cost of higher (private) marginal damage to the common good.

3.2 First best allocation

For this benchmark, consider a benevolent planner with utilitarian social welfare function who is attaching equal Pareto weights λ to both agents:

$$\sum_{\theta} \pi(\theta) \left(\log \left(c_1(\theta) - n_1^{\phi}(\theta) \right) + \log \left(c_2(\theta) - n_2^{\phi}(\theta) \right) \right) \tag{4}$$

The ensuing efficient benchmark allocation where the planner is free to move the resources around, i.e. faces the aggregate resource constraint

$$(\zeta(\theta)) \qquad c_1(\theta) + c_2(\theta) \leq (1 - D(P(\theta)))(\theta_1 n_1(\theta) + \theta_2 n_2(\theta)) \tag{5}$$

is characterized by the following first order conditions:

$$\left[(1 - D(P(\theta)))\theta_i + \frac{\mathrm{d}(1-D(P(\theta)))}{\mathrm{d}n_i(\theta)} (\theta_i n_i(\theta) + \theta_{-i} n_{-i}(\theta)) \right] = \phi n_i^{\phi-1}(\theta) \tag{6}$$

$$\frac{c_1(\theta) - n_1^{\phi}(\theta)}{c_2(\theta) - n_2^{\phi}(\theta)} = 1 \tag{7}$$

Comparing the production assignment condition reveals the well-known result of tragedy of the commons (Gordon, 1954): the self-interested agent fails to internalize the marginal damage caused by his actions to the others. This leads to inefficient use of the common good resource. On the other hand, the planner manages to fully internalize these externalities and so restores economic efficiency.

Moreover, given the utilitarian SWF, agents' idiosyncratic risks are perfectly insured, i.e. their consumption is constant across all the states.

3.3 Introducing voluntary participation constraints

Abstracting from the first best benchmark, there is obviously nothing preventing the agents from getting together to co-operate over the maintenance of the common good and/or risk sharing. However, such a co-operation may be vulnerable to the possibility of some participants defaulting on the associated commitments in order to consume their entire output and/or free ride on the maintenance effort of others. Thus, I introduce now constraints reflecting these concerns.

Ex-post constraint

First of all, I require allocations to satisfy the ex-post limited commitment constraint of the following form:

$$\mu_i(\theta): \quad \log\left(c_i^*(\theta) - (n_i^*(\theta))^\phi\right) \geq V_i^{nc}\left(\theta, \gamma_i(\theta), n_{-i} = n_{-i}^{coop}(\theta)\right) \qquad \forall\theta,\ i \in \{1,2\} \tag{8}$$

where * denotes optimal values in the co-operation and $V_i^{nc}\left(\theta, \gamma_i(\theta), n_{-i} = n_{-i}^{coop}(\theta)\right)$ is the problem as in (1) but taking the labor supply decision of the other agent as given at the level of n_{-i}^{coop} where depending on the type of agreement considered, co-operation is over risk sharing and/or common good ($coop \in \{RS, CG, RS+CG\}$). Denote by $\mu_i(\theta)$ the Lagrangian multiplier on constraint (8).

From now on, let us refer to $1-\gamma_i(\theta) \in [0,1]$ as a reduced-form parameter measuring the planner's punishment severity in case of agent's deviation. Intuitively speaking, the constraint (8) says that if an agent decides to deviate after having learnt his productivity, the planner will be able to destroy share $1-\gamma$ of his net consumption. This translates directly into commitment to co-operate within the agreement as long as in a given state θ each agent gets at least the value he could get by deviating and consuming share $\gamma_i(\theta)$ of his own output. Notice that $\gamma_i(\theta) = 1\ \forall\theta$ corresponds to no-commitment; and if on the other hand $\gamma_i(\theta) = 0\ \forall\theta$, we are back in the full commitment scenario ensuring the first best allocation.

The only reason for modeling limited commitment in such a reduced-form way is to introduce these well-known frictions (as in Kocherlakota (1996) or Kehoe and Levine (1993)) in a simplified framework allowing for clear exposition of main results. The quantitative section with a dynamic version of this model shows that the ideas presented here survive in a more general setting.

Ex-ante constraint

Secondly, I require the co-operation to be desirable from the ex-ante point of view, i.e. to satisfy the participation (or individual rationality) constraint of the form:

$$\eta_i: \quad \sum_\theta \pi(\theta)\log\left(c_i^* - (n_i^*)^\phi\right) \geq \sum_\theta \pi(\theta) V_i^{nc}\left(\theta, 1, n_{-i} = n_{-i}^{coop}(\theta)\right) \tag{9}$$

Denote by η_i the Lagrangian multipliers on (9).

This constraint stipulates that the expected value of co-operating has to be at least as great as the expected value of a unilateral deviation where the deviating agent gets to consume his entire endowment (since here $\gamma_i(\theta) = 1\ \forall\theta$). Additionally, in case of an agreement including common good co-operation, the deviation assumes that the other agent does still maintain the common good, while the deviating agent gets to free ride on his efforts.

The reason for including the ex-ante constraint is to ensure that there is a deep motive for providing insurance within this agreement. To this end, notice that a problem with risk neutral agents, constraint (8) with e.g. $\gamma_i(\theta) = 0\ \forall\theta, i$ and without constraint (9) would still be able to deliver perfect consumption smoothing without the agents really needing it.

3.4 Voluntary risk sharing

Given the discussion above, the centralized version of risk sharing agreement is a solution to maximizing SWF (4) subject to [1] limited commitment and participation constraints (8) and (9); [2] aggregate resource constraint (5); and [3] labor supply decisions at the level of self-interested agents, i.e. without internalizing the production externalities. Formally, this last constraint reads:

$$n_i \textit{ solves } \left[(1 - D(P(\theta)))\,\theta_i + \tfrac{\mathrm{d}(1-D(P(\theta)))}{\mathrm{d}n_i(\theta)}\theta_i n_i(\theta) \right] = \phi n_i(\theta)^{\phi-1}\ \ \forall i \in \{1, 2\} \tag{10}$$

The ensuing allocation can be characterized by the FOC/constraint (10) and the following consumption sharing optimality condition:

$$\frac{c_1(\theta) - n_1^{\phi}(\theta)}{c_2(\theta) - n_2^{\phi}(\theta)} = \frac{\lambda_1 + \eta_1 + \mu_1(\theta)}{\lambda_2 + \eta_2 + \mu_2(\theta)} \tag{11}$$

First of all, with equal Pareto weights and agents being ex-ante identical, we have $\lambda_1 = \lambda_2$ and $\eta_1 = \eta_2$. Now, consider the case of no commitment, i.e. of $\gamma_i(\theta) = 1\ \forall\theta, i$. The ex-ante participation constraint (9) will be always satisfied. Unfortunately this is not true about the limited commitment constraint (8): a uncommitted agent with a relatively higher productivity shock will always find it desirable to deviate and consume his entire output. Thus, the allocation will be characterized by no risk sharing and everyone consuming their own output.

Nonetheless,

Lemma 3.1. *There exists a vector Γ of commitment parameters $\gamma_i(\theta) \leq 1$ s.t. risk sharing with voluntary participation is feasible. In case of a risk sharing agreement, for any $\epsilon(\theta) > 0$ a vector of $\gamma_i(\theta) = 1 - \epsilon(\theta)\ \forall\theta$ leads to a interior risk sharing solution.*

This result follows immediately by continuity and noticing that if on the other hand $\gamma_i(\theta) = 0$, we are in the first best benchmark (i.e. full commitment). For instance, for the case of a risk sharing agreement, a vector with all rows $\gamma_i(\theta) = 1 - \epsilon(\theta)\ \forall\theta$ (with any $\epsilon(\theta) > 0$) will lead to a interior risk sharing solution.

Let us assume for now that

Assumption 3.1: The $\gamma_i(\theta) \in \Gamma$ are s.t. the interior solution exists.

As a consequence, the planner will shift the resources around until either [1] the marginal utilities of consumption are equalized across agents; or [2] the limited commitment constraint (8) for the agent with higher productivity shock is binding.

Finally, notice that the constraint (10) implies that the allocation's efficiency level will coincide with the one of a total autarky. The next allocation is a first step towards improving it.

3.5 Voluntary co-operation over the common good

Similarly to the risk sharing agreement, the centralized version of voluntary co-operation over the common good is a solution to maximizing SWF (4) subject to [1] limited commitment and participation constraints (8) and (9); and [2] individual budget constraints (2) (as opposed to aggregate resource constraint (5), as in the risk sharing agreement).

Then, the solution can be characterized by the following optimality conditions:

$$\frac{c_1(\theta) - n_1^{\phi}(\theta)}{c_2(\theta) - n_2^{\phi}(\theta)} = \frac{\frac{1}{\zeta_1(\theta)}\left(\lambda_1 + \eta_1 + \mu_1(\theta)\right)}{\frac{1}{\zeta_2(\theta)}\left(\lambda_2 + \eta_2 + \mu_2(\theta)\right)} \tag{12}$$

$$\begin{aligned} \phi n_i^{\phi-1}(\theta) = {} & \left(1 - D\left(P(\theta)\right)\right)\theta_i + \frac{\mathrm{d}\left(1 - D\left(P(\theta)\right)\right)}{\mathrm{d}n_i(\theta)} \sum_{j=1}^{2} \frac{\zeta_j(\theta)}{\zeta_i(\theta)} \theta_j n_j(\theta) \\ & - \frac{\eta_{-i}}{\zeta_i(\theta)} \frac{\mathrm{d}V_{-i}^{nc}\left(\theta, \gamma_i(\theta), n_i = n_i^{CG}(\theta)\right)}{\mathrm{d}n_i(\theta)} - \frac{\mu_{-i}(\theta)}{\zeta_i(\theta)} \frac{\mathrm{d}V_{-i}^{nc}\left(\theta, \gamma_i(\theta), n_i = n_i^{CG}(\theta)\right)}{\mathrm{d}n_i(\theta)} \end{aligned} \tag{13}$$

First of all, notice that the net consumption may vary over the states not only due to occasionally binding limited commitment constraints, but also due to different values of Lagrange multipliers on budget constraints.

Furthermore, the optimality conditions (12) and (13) show us that

Proposition 3.2. *In presence of idiosyncratic shocks, an optimal agreement on common good co-operation provides indirect insurance through the production rights assignment.*

To see this, notice that whenever agent's limited commitment constraint binds due to a low assignment of net consumption $c_i(\theta) - n_i^{\phi}(\theta)$ (and so high marginal utility), the Lagrange multiplier $\mu_i(\theta)$ takes on a positive value. Lack of direct risk sharing implies that the value of the associated Lagrange multiplier on agent's i budget constraint $\zeta_i(\theta)$ has to go up in order to balance the increase of $\mu_i(\theta)$ in FOC (12). However, these two need not necessarily balance each other out: the net consumption may actually increase due to adjustments in the production rights. This can be seen in the FOC (13): an increase in $\zeta_i(\theta)$ reduces the weight agent i attaches to its externalities on others, which (ceteris paribus) implies an increase in n_i. In other words, the optimal agreement on common good co-operation in presence of idiosyncratic risk should involve some form of indirect insurance allowing the less productive agents to produce above the efficient level. Conversely, the participation constraint (9) shows us that this value of insurance may facilitate eliciting co-operation among agents.

Notice that the full efficiency would require having $\zeta_1(\theta) = \zeta_2(\theta)$ in (13) - a feature that in general cannot be achieved by this agreement. I show below that an agreement allowing also for risk sharing does provide such a feature.

In the above, I have ignored the last two terms of the FOC (13).[6] In general, they both constitute an attempt at stabilizing the co-operation: whenever the participation constraint of the other agent $-i$ is binding, the planner dictates the agent i to marginally increase his production in order to reduce the value of $-i$'s deviation.

Finally, because deviating and free riding on efforts of others ensures a strictly higher output than deviation in a risk sharing agreement without common good co-operation:

Proposition 3.3. *If the variance of idiosyncratic shocks σ is sufficiently large, the ex-ante participation constraint (9) for the common good co-operation agreement will be satisfied.*

In other words, the higher is the level of idiosyncratic risk in the economy, the easier it becomes to establish common good co-operation. Moreover, the very same observation about the value of deviation being increased by the free riding incentives squared with the limited commitment constraints (8) shows that

Lemma 3.4. *There exists the vector of a minimum required degree of commitment $\bar{\Gamma}$ with $\bar{\gamma}_i(\theta) = 1 - \bar{\epsilon}(\theta)\ \forall\theta$ s.t. the common good co-operation exists.*

In this sense establishing the common good co-operation is more difficult than the risk sharing co-operation.

Relatedly, the limited commitment constraint (8) may now be binding for both the agents with high and low productivity shocks. In particular, this is due to the way the indirect insurance in this setup works by allowing the agent with higher (lower) productivity to reduce (increase) his use of the common good below (above) the efficient level. Obviously, if limited commitment constraints of both agents are binding, there may exist no interior solution to the problem.

3.6 Voluntary risk sharing and common good co-operation

Given the underlying externalities and idiosyncratic risk, a natural solution to the problem seems to be the combination of both institutions allowing for risk sharing and common good co-operation. Such an agreement draws on the advantages of the two previous ones. In particular, the centralized allocation combining both institutions is a solution to the planner maximizing SWF (4) subject to [1] limited commitment and participation constraints (8) and (9); and [2] aggregate resource

[6] Notice also that since we [1] assume ex-ante homogenity; and [2] focus on a commitment vector Γ leading to a interior solution, we can ignore the first term ($\eta_1 = \eta_2 = 0$).

constraint (5) (as opposed to individual budget constraints (2) in the co-operation over the common good only).

First of all, notice that risk sharing facilitates establishing the common good co-operation. Formally:

Proposition 3.5. *For a given vector $\Gamma \leq \bar{\Gamma}$, the set of allocations that can by supported by the joint agreement on both risk sharing and common good co-operation is a strict superset of the corresponding set for the agreement on [1] common good co-operation; and [2] risk sharing.*

This result is a simple consequence of the fact that now the planner faces strictly weaker constraints (see (5) and (2)) as compared to the common good co-operation agreement only. XXX Add intuition/decentralization XXX

Then, the solution to this agreement can be characterized by the following optimality conditions:

$$\frac{c_1(\theta) - n_1^{\phi}(\theta)}{c_2(\theta) - n_2^{\phi}(\theta)} = \frac{\lambda_1 + \eta_1 + \mu_1(\theta)}{\lambda_2 + \eta_2 + \mu_2(\theta)} \tag{14}$$

$$\phi n_i^{\phi-1}(\theta) = (1 - D(P(\theta)))\,\theta_i + \frac{\mathrm{d}(1 - D(P(\theta)))}{\mathrm{d}n_i(\theta)} \sum_{j=1}^{2} \theta_j n_j(\theta) \tag{15}$$

$$- \frac{\eta_{-i}}{\zeta_i(\theta)} \frac{\mathrm{d}V_{-i}^{nc}\left(\theta, \gamma_i(\theta), n_i = n_i^{RS+CG}(\theta)\right)}{\mathrm{d}n_i(\theta)} - \frac{\mu_{-i}(\theta)}{\zeta_i(\theta)} \frac{\mathrm{d}V_{-i}^{nc}\left(\theta, \gamma_i(\theta), n_i = n_i^{RS+CG}(\theta)\right)}{\mathrm{d}n_i(\theta)}$$

Firstly, notice that this allocation is constrained efficient, i.e. the best attainable allocation by the planner facing the deep friction of limited commitment and voluntary participation constraints.

Secondly, comparing FOCs (15) and (13) shows us that

Proposition 3.6. *Complementing the common good co-operation agreement with risk sharing removes the latter's efficiency distortions.*

In other words, risk sharing allows for full alignment of private and social marginal costs. Consequently, this implies that as long as there is an interior solution to the problem, the production rights will be always assigned efficiently. This means that the low (high) productivity agents produce at efficient levels, and receive (share) some of their consumption after the production takes place. Loosely speaking, this dynamic shows us that the two institutions may be seen as complementary. Section 4 below investigates more carefully when this is actually the case.

With the latter in mind and the fact that as we move from the common good to the joint agreement increases the LHS of limited commitment constraints (8) without affecting their RHS, it follows that

Corollary 3.7. *Moving from the common good co-operation to the joint common good and risk sharing agreement allows for a Pareto improvement increasing the total consumption and improving consumption smoothing.*

For moving from the risk sharing only to joint co-operation does also increase the RHS of constraints (8), reversing the logic yields the following

Corollary 3.8. *Moving from the risk sharing co-operation to the joint common good and risk sharing agreement allows for a Pareto improvement with either [1] an improvement in consumption smoothing and increase in total consumption; or [2] deterioration in consumption smoothing compensated by a large enough increase in total consumption.*

Finally and similarly as in the common good agreement, the participation constraints can now be binding for two reasons. Firstly, as before, the agent with high productivity realization may be tempted to leave the contract in order to avoid sharing his output with others. Secondly, the low productive agent (and so the one using common good inefficiently) may want to increase his labor supply above the agreement-specified level in order to smooth the impact of his bad shock. As a comparison, a standard model with production and no common good would not generate such a dynamic: when low productivity agents increase their labor supply, this imposes no externalities and so does not make a difference to others.

This result is related to Genicot and Ray (2003) who showed that allowing for group deviations (as opposed to unilateral as is standard in literature and also here) implies existence of an upper bound on the coalition size above which the risk sharing contract has no co-operative solution. While maintaining the assumption of unilateral deviations, the interaction of risk sharing with common good externalities constitutes a potential reason for making the limited commitment constraints of both agents binding and as such it may be a reason why in some cases coalitions are unlikely to arise. This effect might be particularly strong in an environment with many agents when the free riding incentives of small individuals become particularly large (see e.g. Kolstad and Toman (2001)).

3.7 Decentralizations [in progress]

Three options using state-dependent Arrow(-Debreu) securities:

1. enhanced by action-dependancy,
2. together with a pigouvian taxation system,
3. together with a cap-and-trade mechanism.

See Appendix B for the decentralizations of a full dynamic model.

4 Complementarity and substitution between risk sharing and common good

As already mentioned, a quick look on the optimal production rights assignment conditions in presence of common good co-operation without (Section 3.5, eq. (13)), and with risk sharing (Section 3.6, eq. (15)) shows that the two institutions may be complementary to each other. To be more precise though, this sections formally investigates when the two institutions of risk sharing and solving the common good problem may be complementary or substitutable to each other. The following definition operationalizes the notion of complementarity and substitution from now on:

Definition 4.1. Risk sharing and solving the common good problem are complementary to each other if

$$\sum_{i=1}^{2} u^{RS}\left(c_i\right) - \sum_{i=1}^{2} u^{nc}\left(c_i\right) < \sum_{i=1}^{2} u^{RS+CG}\left(c_i\right) - \sum_{i=1}^{2} u^{CG}\left(c_i\right) \tag{16}$$

When this inequality is reversed, the two are substitutes to each other.

Intuitively, complementarity (substitution) means that presence of the common good externalities improves (impedes) the risk sharing possibilities, or conversely that risk sharing makes sustaining agreements aimed at fixing common good externalities less (more) difficult.

A first and immediate case when the two institutions are complementary is whenever the associated free-riding incentives are high enough and/or the level of commitment is low enough leading to the collapse of any common good co-operation. In such a case, $\sum_{i=1}^{2} u^{CG}\left(c_i\right) = \sum_{i=1}^{2} u^{nc}\left(c_i\right)$ and since $\sum_{i=1}^{2} u^{RS}\left(c_i\right) < \sum_{i=1}^{2} u^{RS+CG}\left(c_i\right)$ is always true, the complementarity follows.

Another case when the two institutions are complementary is when the efficiency losses associated with indirect redistribution in the common good co-operation are large enough such that $\sum_{i=1}^{2} u^{CG}\left(c_i\right)$ is close enough to $\sum_{i=1}^{2} u^{nc}\left(c_i\right)$.

Otherwise, if the overall output losses due to this indirect redistribution are not excessive, the two institutions are substitute to each other. Importantly, this does not mean that introducing risk sharing to a common good co-operation will reduce the welfare (again, $\sum_{i=1}^{2} u^{RS}\left(c_i\right) < \sum_{i=1}^{2} u^{RS+CG}\left(c_i\right)$ always holds true). It rather means that the direct mechanism of risk sharing replaces the indirect insurance role of the common good co-operation without adding too much of efficiency gains for it.

XXXX: Mathematically, this The source of these inefficiencies are the wedges $\frac{\zeta_j(\theta)}{\zeta_i(\theta)}$. Give a numerical example plots.

5 Empirical application of the model: farming in India

The purpose of this Section is to provide some suggestive evidence that interaction between the two institutions studied in this paper may actually be important in case of Indian villages that are not only relying on informal risk sharing,[7] but are also engaging into co-operation intensive rice cultivation. Then, I use this evidence to develop in Section 6 the quantitative version of the model in order to understand the productivity and insurance gains stemming from this interaction.

In particular, this section first argues that cultivating rice requires a lot of co-operation among farmers in order to achieve high yields. Although the rice farming example may be arguably a sensible application of the theoretical model, this section outlines an empirical test of its applicability.Then, I present two hypotheses that allow me to test the applicability of the theory developed above to the case of farming. Finally, I validate the testable hypotheses using the ICRISAT data set and so demonstrate that [1] the model proposed here can be meaningfully applied to understanding rice farming in developing countries; and [2] that these farmers may be actually benefitting from the interaction between the two institutions studied in this paper.

The example of farming in developing countries is particularly important for two reasons. Firstly, according to UN DESA (2013), population in lowest and low/middle- income countries is projected to grow from 0.9 and 5.9b to 1.8 and 8.2b. At the same time, most of the world's poor live in dry areas with water supply problems. Therefore, we need a good understanding of common good co-operation determinants that can help improve food and water supplies security in these areas.

Secondly, as self-insurance of people living in developing countries is often very limited, the idiosyncratic risks may have devastating effects on household resources. It is therefore of a particular importance to understand the effects of (and insurance against) these shocks.

5.1 Rice farming as an example of common good co-operation

Rice farming is well known to require a lot of co-operation among farmers. For instance, a more recent Science article by psychologists Talhelm et al. (2014) provides so called "the rice theory of culture" which states that the societies cultivating rice are in general much more interdependant due to the higher degree of required co-operation in agriculture (as opposed to e.g. wheat which is much easier to cultivate independently). The dimensions for possible co-operation are manifold.

First of all, growing rice requires large amounts of water. If the rain supplies are insufficient, farmers have to often co-operate over construction, usage and maintenance of additional water tanks and/or irrigation systems.

Furthermore, during the growing period, rice fields - or the so called 'paddies' - have to be consistently covered with the right amount of water. This means that the neighboring pieces of land have to be separated from each other so that the water remains in the farmer's own paddy. Quality of these so called separating 'bunds' is obviously highest if both owners of neighboring fields maintain them. Similarly, if the fields are situated e.g. next to a river, such bunds are

[7] Mazzocco and Saini (2012) document that in Aurepalle the real per-capita transfers given and received are on average 28.3% and 21.1% of non-durable expenditures.

needed for protection of all the fields from flooding.

Additionally, rice often grows in uneven areas, e.g. with terraces. In such cases the neighbors have incentives to co-operate over leveling their fields so that the water spillovers do not lead to crop and mineral material erosion.

Last but not least, cultivating rice is very labor intensive. For this reason farmer-families often synchronize their growing activities so that they are able to help each other through labor sharing.

Needless to say, all of these activities are prone to free riding on efforts of others, and the best outcomes can be only achieved if everyone does take into account the true social cost and/or benefit of their actions.

To expose the common good effects due to rice farming, I will use the control group of non rice farming households. In the data set presented below, this group will be mostly engaged into cultivation of castor, sorghum, pearl millet and pigeon pea or pursuing jobs in e.g. agriculture, black- and goldsmithy, livestock rearing, pot-making, shop-keeping, teaching, trading or weaving. The main point is that villagers involved into cultivating these crops or employed in these professions have far less (if any) of underlying common goods requiring as high a degree of co-operation as rice farming. In fact, this is the very same idea as pursued by Talhelm et al. (2014) who use the wheat farming regions as the control group for the rice farming ones, exactly due to wheat's significantly lower irrigation- and labor-intensity.

5.2 The ICRISAT dataset

The application of the model is based on the panel data on farmers in rural India from ICRISAT's Village Level Studies (VLS). The original dataset covered six locations in rural India starting from July 1975. Each village sample was divided into 4 groups of 10 households each representing families in four land holding classes according to the land size. The literature[8] follows Townsend (1994) by focusing on the 3 villages: Aurepalle, Shirapur and Kanzara.

Figure 1 (adapted from Mahajan et al. (2017)) below shows the geographical location of the three villages and the major rice growing regions in India for two major growing seasons (monsoon Kharif and post-monsoon Rabi). Because the other two villages lie in regions not conductive to growing rice, the focus of this paper will be on Aurepalle.[9]

In order to compare the rice vs. non-rice farmers, I need to divide the households by their agricultural activities. To this end, I firstly classify each datapoint into one of the two major agricultural seasons in India: Kharif (May-October) and Rabi (November-April). Then, using the Plot Cultivation Schedule to determine whether a given household is engaged into rice farming in a given season, I classify a household as a "rice farming household" if the household has been

[8] See e.g. Ligon, Thomas and Worrall (2002), Mazzocco and Saini (2012) or Abraham and Laczo (2017).

[9] XXX: put the appendix reference to show that aggregate risks (in form of rainfall deviations from the mean) are different among 3 villages thus the consumption smoothing comparisons across villages need not be in line with Hypothesis 1.

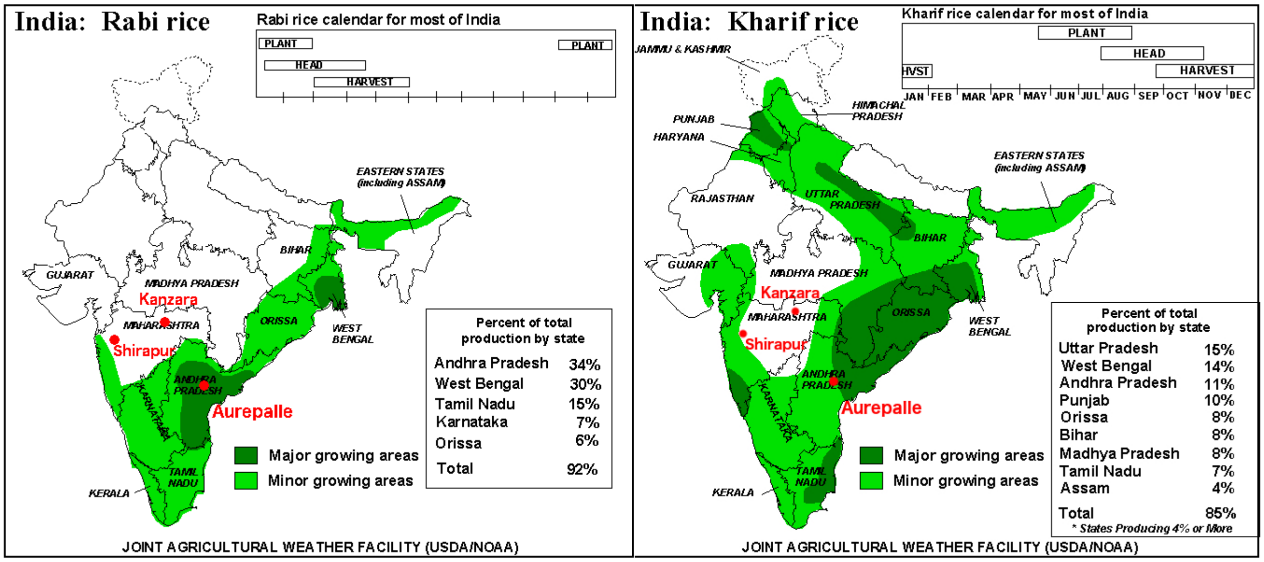


Figure 1: VLS villages and rice growing regions in India (from Mahajan et al. (2017))

producing rice in at least 8 of the 16 agricultural seasons covered by the sample period. As a result, I end up with 11 rice farming and 20 non-rice farming households. Nota bene, 4 of the non-rice farming households had tried cultivating rice in 2-3 seasons but have abandoned this activity for the sake of other crops.

The sample covers the time between July 1975 and December 1983, a total of 102 periods. I drop households with fewer than 80 data points. This implies I am left with 31 households with a total headcount of 252 (as of December 1983).

The VLS records data on production, labor supply, assets, price of goods, rainfall, monetary and non-monetary transaction, household size, age, education from 1975 to 1985. Because of concerns with the accuracy of consumption measurement, I focus on the years 1975-1983. Townsend (1994) gives a detailed description of the data. Thus, I discuss here only the issues specific to this paper.

In order to provide empirical support for the mechanism studied, I need data on consumption expenditures, household income, demographic variables and agricultural activity. The ICRISAT collects information on these variables approximately every month[10] and so the risk-sharing will be estimated using monthly data. I will now discuss how these variables are constructed.

The calculation of monthly consumption and income follows Mazzocco and Saini (2012). The non-durable consumption is the sum of expenditures on milled grain, oil, animal products, fruits and vegetables, and on other non-durable goods such as electricity, water charges, cooking fuels for household use, and expenses for domestic work.

Then, the household's full income is constructed using the household budget constraint, i.e. as

[10]This is approximately since the frequency of the interviews varies and the dates of the interviews differ across households. In order to overcome this problem, I follow Mazzocco and Saini (2012) and for each interview that covers two months I compute the percentage of days that belongs to each month. Then, I assign to each month the corresponding expenditure using this percentage.

total expenditure minus the resources borrowed from different sources, plus the resources saved in different accounts or lent to various individuals, plus the transfers given out, minus the transfers received, plus taxes. The ICRISAT data contains information on all these variables except the cash savings not deposited with a financial institution. The resulting measure of risk sharing should therefore control for households' savings.

In order to control for the household's age and size composition, the consumption and income variables are transformed into per-capita using an age-gender weight as in Townsend (1994).[11] Furthermore, they are both deflated using the consumer price index for agricultural laborers published by the Labour Bureau of India.

The vector of observable demographic-heterogeneity variables comes from Households Member Schedule and is composed of the mean age of adult household members, the number of infants and the age-gender weight.

Finally, when addressing some of the endogeneity issues below, I exploit the policy discontinuity affecting transaction prices relevant only for the rice farmers. To this end, I use the Household Transaction Schedule recording the quantities, dates and prices at which the households sold their crop. Reports on the policy discontinuity come from archives of the Times of India and the Financial Times (see below).

5.3 Test of the theoretical model

The first testable hypothesis follows from the Proposition 3.3 based on the (ex-ante) participation constraint (9) saying that the higher is the variance of idiosyncratic productivity shock risk, the more feasible becomes the common good co-operation. Therefore, we have:

Hypothesis 1: Farmers facing higher idiosyncratic risk will engage into rice cultivation.

The second hypothesis follows from the Corollary 3.8 based on the ex-post limited commitment constraint (8). Again, the very same constraint (adjusted accordingly for the other agent's deviation behavior n_{-i}) is used in both the [1] common good only, [2] risk sharing only, and [3] the joint common good and risk sharing co-operation. Consider, as in Corollary 3.8, the dynamic of enhancing a risk sharing only agreement by the common good co-operation. On its impact, the LHS of the constraints (8) for all state realizations (weakly) increases due to optimization of the common good's use. Holding the RHS constant, this implies that the planner can now improve consumption smoothing in the economy. This leads to:[12]

[11] The age-gender weight is computed by adding the following numbers: for adult males, 1.0; for adult females, 0.9; for males aged 13-18, 0.94; for females aged 13-18, 0.83; for children aged 7-12, 0.67; for children aged 4-6, 0.52; for Toddlers 1-3, 0.32; and for infants 0.05.

[12] Since validating the condition on free riding is empirically very challenging, one of the purposes of developing the quantitative model is to 'reverse-validate' it. In particular, a calibrated dynamic model will be used to show that the consumption smoothing improves as we extend the risk sharing agreement by the common good co-operation.

Hypothesis 2: If the associated free riding incentives are not too large, rice farmers will have better consumption smoothing due to the internalization of common good externalities.

Notice that this hypothesis might be true due to either rice-farmers [1] facing a more volatile idiosyncratic risk process (see Alvarez and Jermann (2000)); or [2] benefiting from the internalization of the common good externalities (which is at the center of the theory developed in this paper). I will address this issue in the Section with empirical results below.

The rest[13] of this section is devoted to [1] describing the dataset used; and [2] the process of testing these hypotheses.

5.4 Empirical results

Hypothesis 1

In order to measure idiosyncratic risk of income, I follow the method proposed by Storesletten et al. (2004). In particular, I use the following decomposition of household's i real per-capita earnings:

$$y_{i,t} = \alpha_0 + \alpha_1 X_{i,t} + \tilde{\epsilon}_{i,t} \tag{17}$$

where $X_{i,t}$ is capturing the aggregate and household-specific determinants of income, and so consequently the $\tilde{\epsilon}_{i,t}$ measures the idiosyncratic risk of a household.

In particular, $X_{i,t}$ contains [1] controls for household characteristics such as: household size, mean age of adults, number of infants, landholding class[14] and household-farming season (i.e. Rabi or Kharif) fixed effects; and [2] the village-time fixed effects. The household-season fixed effects are supposed to control for the type of agricultural activity chosen by the farmer, so that the the resulting measure of idiosyncratic risk is not contaminated by selection of farmer's into occupation. On the other hand, the village-time fixed effects are to control for the aggregate risk specific to the whole village.

In order to allow for some persistence of the shocks, I model the idiosyncratic risk component of group $j \in \{rice,\ non-rice\}$ as an AR(1):

$$\tilde{\epsilon}^j_{i,t} = \rho\tilde{\epsilon}^j_{i,t-1} + e^j_{i,t} \tag{18}$$

[13]Notice that Corollary 3.7 yields a third testable hypothesis saying that conditional on a given level of common good co-operation, the degree of risk sharing should positively predict the households' income. I do not test this hypothesis for the lack of direct data on common good co-operation in the ICRISAT's dataset making differentiation of the degree of common good co-operation impossible. In such a case, focusing on only 11 rice farming households and their reliance on risk sharing transfers (and assuming that their level of common good co-operation is the same) is a futile exercise.

[14]The ICRISAT dataset disaggregates the categories of land ownership into four categories of: 1) laborer (i.e. no land); 2) small farmer; 3) medium farmer; and 4) large farmer.

	ρ	$sd\left(e_t^j\right)$
rice	0.62	$3.03 \cdot 10^9$
non-rice	0.54	$1.16 \cdot 10^9$

Table 1: Estimation results of idiosyncratic risk for households in Aurepalle

The results of estimation in Table 1 validate hypothesis 1. In particular, the standard deviation of the innovation process of rice farmers is 2.6 times larger with somewhat similar persistence. Notice that as I controlled for the household-season fixed effects, this difference should not be seen as a consequence of farmers' selection into occupations. The later part of this Section provides some additional RD-evidence supporting this result.

Hypothesis 2

I run a standard test of consumption smoothing as in Mazzocco and Saini (2012) adapted for the context of this paper. More precisely, the hypothesis is tested using:

$$\Delta exp_{i,t} = \alpha_0 + \alpha_1 \Delta y_{i,t} + \alpha_2 rice_i + \alpha_3 rice_i \cdot \Delta y_{i,t} + \alpha_4 \Delta exp_t^{vil} + \alpha_5 \Delta X_{i,t} + \alpha_6 \Delta \sigma_{t,t+1}^j + \epsilon_{i,t} \quad (19)$$

where exp are the household's i consumption expenditures, y is the household income, $rice$ is the indicator variable whether household i is a rice or non-rice farming, exp^{vil} is the average village consumption expenditures (in order to control for risk sharing's response to aggregate shocks), X contains the same set of controls for household characteristics (as explained above for Hypothesis 1), and variable $\Delta\sigma_{t,t+1}^j,\ j \in \{rice,\ non-rice\}$ is controlling for differing household expectations about the future idiosyncratic productivity risk. Thus, the variable $\Delta\sigma_{t,t+1}^j$ is a difference between periods $t+1$ and t of a group (rice or non-rice) one year (i.e. between periods t and $t+11$) average realized future variance of the innovations $e_{i,t}$ in the AR(1) process(18). Moreover, in order to control for different seasonality patterns between rice and non-rice farming households, I consider two versions of regression (19): without and with household-time fixed effects. Results of estimation are presented in Table 2.

The coefficients of main interest are α_1 on $\Delta y_{i,t}$ and α_3 on the interaction term $rice_i \cdot \Delta y_{i,t}$. In line with other papers in the literature, perfect consumption smoothing is rejected for both groups. More importantly though, the results are in line with Hypothesis 2 saying that the rice farming households have relatively better consumption smoothing. Importantly, the results are robust to different seasonality patterns between households, for instance due to different timing of harvesting or crop's storability. The fact that results do not change significantly might reflect the fact that the latter issue is also addressed by the construction of the households' income variable that already accounts for savings (including those from farming activities).

Dep. var: $\Delta exp_{i,t}$	No FE	FE
$\Delta y_{i,t}$	0.237**	0.238**
	(0.0090)	(0.0091)
$rice_i \cdot \Delta y_{i,t}$	-0.226**	-0.227**
	(0.0097)	(0.0098)
$\Delta \sigma^j_{t,t+1}$	$7.08 \cdot 10^{-10}$**	$7.10 \cdot 10^{-10}$**
	$(3.43 \cdot 10^{-10})$	$(3.43 \cdot 10^{-10})$
$rice_i$	2.854**	3.281**
	(1.305)	(1.329)

Note: Standard errors in parentheses. (**) and (*) indicate that the coefficient is significant, respectively, at the 5 and 10 percent level.

Table 2: Estimation results of consumption smoothing for households in Aurepalle

Similarly, since the expectations of future idiosyncratic risk are controlled for, better consumption smoothing of rice farmers should not be seen as a sole consequence of this group facing higher idiosyncratic risk. Thus, the difference is at least to some extent due to the internalization of common good externalities. Importantly, the seemingly low coefficient on $\Delta \sigma^j_{t,t+1}$ should not be seen as economically insignificant as one standard deviation of $\Delta \sigma^j_{t,t+1}$ amounts to $2.302 \cdot 10^9$ for rice farmers and to $1.07 \cdot 10^9$ for non-rice farmers.

Additional policy discontinuity evidence

Since 1965, each year in autumn the Indian government announces the so called Minimum Support Price (MSP) for some agricultural products. This market intervention guarantees a minimum crop price for which the producers may always sell their products to government agencies and as such provides insurance for agricultural producers against any sharp fall in farm prices. Given its nature, the MSP may often not be binding if it is below the prevailing market price.

As of today, the MSP is announced for many commodities such as cereals, pulses, oilseeds, cotton etc. However, initially MSP was targeted at paddy rice and wheat, with a successive addition of other crops over time.

To this end, Figures 3_XXX and 4_XXX in Appendix A presents two pieces of news from the 1st of September and the 4th of October 1982 in the Financial Times and the Times of India about the announcement and implementation (respectively) of a 6% MSP increase for paddy rice. Importantly, these and other news from the historical archives of both these newspapers do not contain any information about an increase in MSP of any other crop.[15] Thus, to the best of my knowledge, in the season 1982-1983 the MSP increase was pertaining only to paddy rice and as such this policy change has not affected any other type of households apart from the rice farming

[15] I have resorted to data collection through historical archives as the official government data on MSP goes only until early 2000s (see e.g. http://eands.dacnet.nic.in/MSP.htm or http://farmer.gov.in/mspstatements.aspx)

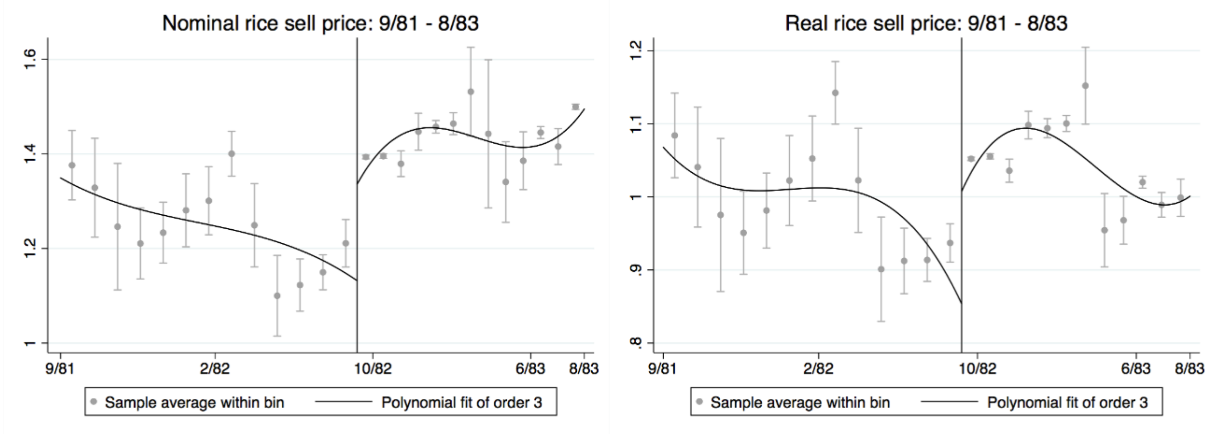


Figure 2: Rice-crop sell price in Aurepalle: 09/1981-08/1983

ones.[16]

The impact of the policy change is clearly visible in the transaction data in Aurepalle. In particular, Figure 3 shows nominal and real paddy rice sell prices for the year before and year after the policy change announcement. Both of these convey the same evidence on the real impact of the policy on the prices faced by farmers in Aurepalle.[17] The goal is now to use this policy change to provide some additional evidence for Hypotheses 1 and 2.

Returning to Hypothesis 1 stating that households facing higher idiosyncratic risk are more likely to become rice farmers, the dataset reveals that the policy change does not induce any of the Aurepalle's non-rice farming households to try cultivating rice in the season of and after the announcement. Since the policy reduces the risk associated with growing rice, this suggests that households do not respond to risk characteristics of a chosen crop.

A potential explanation to it may be that there are some significant fixed costs curbing the response of farmers. However, recall from Section 5.2 the way the households are categorized into the two rice and non-rice farming groups. In particular, there are 4 of the non-rice farming households that had tried cultivating rice in 2-3 seasons but have abandoned it afterwards. Although these households had some experience, know-how and equipment allowing for cultivation of rice, none of them responded to the policy change, suggesting that this barrier need not be pivotal.

As far as it concerns the Hypothesis 2 on consumption smoothing of both household groups, I re-run regression (19) with fixed effects separately for rice and non-rice households one year before

[16]This is also confirmed by Figure XXX in Appendix XXX which shows that in September 1982 there has been no significant discontinuity in transaction prices of other major crops in Aurepalle (such as XXX).

[17]Notice that the post-announcement prices are fluctuating somewhat above the prices reported in the news in Figures 3_XXX and 4_XXX (1.35-1.50 v.s. the MSP of 1.22). Mind, however, that there are different quality classes of paddy rice and the price reported in the news in Figures 3_XXX and 4_XXX is pertaining the common quality of the rice. Higher classes of rice were often found (among others) in Aurepalle's macro-region Andhra Pradesh (see Figure 5_XXX in Appendix A) and were usually given a higher MSP (unfortunately not reported in the news in Figure 3_XXX and 4_XXX). Nonetheless, see [1] Figure 5_XXX in Appendix A for historical evidence from Times of India on September 30, 1978 showing that superfine rice would get a premium of 21%; and [2] http://farmer.gov.in/mspstatements.aspx for evidence of differential pricing based on quality today.

Dep. var: $\Delta exp_{i,t}$	09/1981 - 08/1982	09/1982-08/1983
$\Delta y_{i,t}^{rice}$	0.003	0.055**
	(0.017)	(0.017)
$\Delta y_{i,t}^{non-rice}$	0.332**	0.299**
	(0.026)	(0.055)

Note: Standard errors in parentheses. (**) and (*) indicate that the coefficient is significant, respectively, at the 5 and 10 percent level.

Table 3: Estimation results of consumption smoothing for households in Aurepalle around the date of policy change

and one year after the discontinuity. The results of this exercise are in Table 3.

On impact of the increase in MSP of paddy rice, the consumption smoothing of rice farming households deteriorates, while the consumption smoothing of the non-rice farming households remains intact (the difference in coefficients on $\Delta y_{i,t}^{non-rice}$ is statistically insignificant). This evidence tells us that first of all the two groups of households are truly different and so the adoption of the "rice theory" from Talhelm et al. (2014) in this context does make sense.

Secondly, the evidence can be reconciled theoretically as a rational reduction in the desire for consumption smoothing upon a persistent positive wealth shock with households having CRRA preferences. Thus, this additional evidence may be used below in the quantitative model while [1] choosing the class of utility function; and [2] calibrating the model so that the households exhibit behavior similar to the one presented here.

6 Quantitative model

6.1 Setup

Consider a dynamic infinite-horizon economy with 2 rice farming households generalized in the following way. Households are risk averse, dislike providing labor supply according to the utility function $u(c,n)$ and are hand-to-mouth. They discount future at the rate of β. Moreover, they are ex-ante identical and become ex-post heterogenous due to idiosyncratic i.i.d. productivity shocks$\theta_{i,t} \in \Theta = \left(\theta^1, ..., \theta^{N_\theta}\right)$, $0 < \theta^1 < ... < \theta^{N_\theta} < \infty$ following transition matrix Π with a strictly positive probability for realization of all productivity levels and the two first (long-run) moments of $\mu = \mathrm{E}(\theta)$ and $\sigma^2 = \mathrm{Var}(\theta)$. Again, let $\theta_t \equiv \{\theta_{1,t}, \theta_{2,t}\}$ denote the state of all productivity shock realizations and let superscript variables denote history, e.g. with θ^t denoting the history of shock realizations. The shocks are assumed to be markovian and so we have that $\Pi\left(\theta_t|\theta^{t-1}\right) = \Pi\left(\theta_t|\theta_{t-1}\right)$.

The common good can be now thought of as the state of all the investments in the rice fields such as: [1] the state of irrigation systems; [2] the amount of water in tanks or other forms of storage; [3] the state of bunds protecting the fields from water over-flows; or [4] the land's level leading not only to erosion of one's piece of land but also of the neighboring fields. With this in mind, the common good is now a state variable meaning that the past actions do affect it's current status.

In particular, the pollution variable P_t is a stock variable entering the status of the common good in the following way $1 - D(P_t) = \exp(-\Phi P_t)$ with $P_t = n_{1,t}^{\kappa}(\theta) + n_{2,t}^{\kappa}(\theta) + \phi P_{t-1}$, where ϕ is the decay rate of past (mal-)investments' effects on the common good; $\kappa \geq 1$ governs the rate at which the damages accumulate; and Φ is used to scale the damage function. These parametrical assumptions are largely in line with the ones employed by Golosov et al. (2014).

Finally, it is well understood that within the class of models where the endogenously incomplete market is generated by the limited commitment friction, it is possible to support a continuum of different equilibria with properties depending on the exact specification of the outside option. In what follows, I will focus on the equilibrium with the highest supportable degree of risk sharing. This corresponds to assuming a full retaliation strategy, i.e. that whenever household i deviates, they will be able to free-ride on the efforts of household $-i$ for one period only as after this the latter will also deviate to their Nash equilibrium strategy, and so from the second period onwards the outside option will coincide with the Nash equilibrium (and lead to the associated tragedy of the commons). The recursive formulation below makes all of this precise.

6.2 Recursive formulation

I will now provide a recursive formulation of outside option and the constrained efficient contract with both risk sharing and common good co-operation that will be used for the quantitative analysis. Given this, I will later highlight the differences between different variants of allocations without either of risk sharing or common good agreement.

Risk sharing and common good co-operation

The value of the constrained efficient contract involving both risk sharing and common good co-operation is given by a solution of a benevolent planner's problem facing the resource and limited commitment (in the spirit of the model in Section 2; see below for details) constraints. The relevant state variables of the individual household for the co-operative contract are $x = (\mu_{1,t}, \mu_{2,t}, \theta_t, P_t) \in \mathrm{R}_+ \times \mathrm{R}_+ \Theta \times \mathrm{R}_+$, where μ's are the Lagrange multipliers on the relevant limited commitment constraints.

$$\max_{\{c_{i,t}, n_{i,t}\}_{t\geq 0,}} \sum_{t=0}^{\infty} \sum_{\theta^t} \lambda_i \beta^t \pi\left(\theta^t\right) u\left(c_{i,t}, n_{i,t}\right) \tag{20}$$

subject to:

$$\left(\zeta\left(\theta^{t}\right)\right)\ \sum_{i=1}^{2} c_{i,t} \leq \left(1 - D\left(P_t\left(\theta^{t}\right)\right)\right) \sum_{i=1}^{2} \theta_{i,t} n_{i,t}\ \forall t \tag{21}$$

$$\left(\mu_i\left(\theta^{t}\right)\right)\ \sum_{s=t}^{\infty}\sum_{\theta^s} \beta^{s-t}\pi\left(\theta^{s}|\theta^{t}\right) u\left(c_{i,t}, n_{i,t}\right) \geq V_{i,s}^{nc}\left(\theta^{s}\right)\ \ \forall i\ and\ s \geq t \tag{22}$$

Using the methodology developed by Marcet and Marimon (1998/2017), the Lagrangian associated with problem (20) can be re-written as a saddle-point functional equation:

$$V_{i,s}^{RS+CG}\left(\mu_{1,s}, \mu_{2,s}, \theta_s, P_s\right) = \inf_{\mu_i \geq 0} \sup_{\{c_{i,s}, n_{i,s}\}_{s\geq t}} \sum_{i=1}^{2}\sum_{s=t}^{\infty}\sum_{\theta^t} \beta^{s-t}\pi\left(\theta^{s}\right)$$
$$\left[M_i\left(\theta^{s-1}\right) u\left(c_{i,s}, n_{i,s}\right) + \mu_i\left(\theta^{s}\right)\left[u\left(c_{i,s}, n_{i,s}\right) - V_{i,s}^{aut}\left(\theta^{s}\right)\right]\right]$$
$$+\sum_{s=t}^{\infty}\sum_{\theta^t} \beta^{s-t}\pi\left(\theta^{s}\right)\zeta\left(\theta^{s}\right)\left[\left(1 - D\left(P_s\left(\theta^{s}\right)\right)\right)\sum_{i=1}^{2}\theta_{i,s} n_{i,s} - \sum_{i=1}^{2}\left(c_{i,s}\right)\right]$$

Assuming GHH preferences, the associated first order conditions for period $s \geq t$ read:

$$\begin{aligned}
c_{i,s:}\quad \frac{1}{c_{i,s} - \gamma n_{i,s}^{\phi}}\left[M_i\left(\theta^{s-1}\right) + \mu_i\left(\theta^{s}\right)\right] &= \zeta\left(\theta^{s}\right)\\
n_{i,s}:\quad \frac{1}{c_{i,s} - \gamma n_{i,s}^{\phi}}\gamma\phi n_{i,s}^{\phi-1}\left[M_i\left(\theta^{s-1}\right) + \mu_i\left(\theta^{s}\right)\right] &= \zeta\left(\theta^{s}\right)\left(1 - D\left(P_s\left(\theta^{s}\right)\right)\right)\theta_{i,s}
\end{aligned}$$
$$+\sum_{s'=s}^{\infty}\sum_{\theta^{s'}} \beta^{s'-s}\pi\left(\theta^{s'}\right)\zeta\left(\theta^{s'}\right)\frac{\mathrm{d}\left(1 - D\left(P_{s'}\left(\theta^{s'}\right)\right)\right)}{\mathrm{d}n_{i,s}}\sum_{j=1}^{2}\theta_{j,s'} n_{j,s'} \ - \ \sum_{s'=s}^{\infty}\sum_{\theta^{s'}} \mu_i\left(\theta^{s'}\right)\frac{\partial V_{i,s'}^{aut}\left(\theta^{s'}\right)}{\partial n_{i,s}}$$
$$-\sum_{s'=s}^{\infty}\sum_{\theta^{s'}} \mu_{-i}\left(\theta^{s'}\right)\frac{\partial V_{-i,s'}^{aut}\left(\theta^{s'}\right)}{\partial n_{i,s}}$$

Following Kehoe and Perri (2002), let's define new variables:

$$v_i\left(\theta^{t}\right) = \frac{\mu_i\left(\theta^{t}\right)}{M_i\left(\theta^{t}\right)} \Rightarrow 1 - v_i\left(\theta^{t}\right) = \frac{M_i\left(\theta^{t-1}\right)}{M_i\left(\theta^{t}\right)} \tag{23}$$

$$z\left(\theta^{t}\right) = \frac{M_2\left(\theta^{t}\right)}{M_1\left(\theta^{t}\right)} = \frac{1 - v_1\left(\theta^{t}\right)}{1 - v_2\left(\theta^{t}\right)} z\left(\theta^{t-1}\right) \tag{24}$$

$$\tilde{v}_i\left(\theta^{t}\right) = \frac{\mu_{-i}\left(\theta^{t}\right)}{M_i\left(\theta^{t}\right)} \Rightarrow \begin{cases} z\left(\theta^{t}\right) - \tilde{v}_i\left(\theta^{t}\right) = \frac{M_{-i}\left(\theta^{t-1}\right)}{M_i\left(\theta^{t}\right)} & if\ i = 1\\ \frac{1}{z\left(\theta^{t}\right)} - \tilde{v}_i\left(\theta^{t}\right) = \frac{M_{-i}\left(\theta^{t-1}\right)}{M_i\left(\theta^{t}\right)} & if\ i = 2 \end{cases} \tag{25}$$

Now we can use (23)-(25) to rearrange the first order conditions in the following way:

$$
\begin{aligned}
c_{i,s}: \quad & \frac{c_{2,s}-\gamma n_{2,s}^{\phi}}{c_{1,s}-\gamma n_{1,s}^{\phi}} = z\left(\theta^{t}\right) = \frac{1-v_{1}\left(\theta^{t}\right)}{1-v_{2}\left(\theta^{t}\right)} z\left(\theta^{t-1}\right) \\
n_{i,s}: \quad & \frac{1}{c_{1,s}-\gamma n_{1,s}^{\phi}} \gamma \phi n_{i,s}^{\phi-1} = \frac{1}{c_{1,s}-\gamma n_{1,s}^{\phi}} \left(1-D\left(P_{s}\left(\theta^{s}\right)\right)\right) \theta_{i,s} \\
& + \sum_{s'=s}^{\infty} \sum_{\theta^{s'}} \beta^{s'-s} \pi\left(\theta^{s'}\right) \frac{M_{i}\left(\theta^{s'}\right) U_{ic}\left(\theta^{s'}\right)}{M_{i}\left(\theta^{s}\right)} \frac{\mathrm{d}\left(1-D\left(P_{s'}\left(\theta^{s'}\right)\right)\right)}{\mathrm{d} n_{i,s}} \sum_{j=1}^{2} \theta_{j,s'} n_{j,s'} \\
& - \sum_{s'=s}^{\infty} \sum_{\theta^{s'}} \frac{\mu_{i}\left(\theta^{s'}\right)}{M_{i}\left(\theta^{s}\right)} \frac{\partial V_{i,s'}^{aut}\left(\theta^{s'}\right)}{\partial n_{i,s}} \\
& - \sum_{s'=s}^{\infty} \sum_{\theta^{s'}} \frac{\mu_{-i}\left(\theta^{s'}\right)}{M_{i}\left(\theta^{s}\right)} \frac{\partial V_{-i,s'}^{aut}\left(\theta^{s'}\right)}{\partial n_{i,s}}
\end{aligned}
$$

Rearranging yields:

$$
\begin{aligned}
c_{i,s}: \quad & \frac{c_{2,s}-\gamma n_{2,s}^{\phi}}{c_{1,s}-\gamma n_{1,s}^{\phi}} = z\left(\theta^{t}\right) = \frac{1-v_{1}\left(\theta^{t}\right)}{1-v_{2}\left(\theta^{t}\right)} z\left(\theta^{t-1}\right) \\
n_{i,s}: \quad & \frac{1}{c_{1,s}-\gamma n_{1,s}^{\phi}} \gamma \phi n_{i,s}^{\phi-1} = \frac{1}{c_{1,s}-\gamma n_{1,s}^{\phi}} \left(1-D\left(P_{s}\left(\theta^{s}\right)\right)\right) \theta_{i,s} \\
& + \sum_{s'=s}^{\infty} \sum_{\theta^{s'}} \beta^{s'-s} \pi\left(\theta^{s'}\right) \frac{U_{ic}\left(\theta^{s'}\right)}{\left[\Pi_{j=s}^{s'}\left(1-v_{i}\left(\theta^{j}\right)\right)\right]} \frac{\mathrm{d}\left(1-D\left(P_{s'}\left(\theta^{s'}\right)\right)\right)}{\mathrm{d} n_{i,s}} \sum_{j=1}^{2} \theta_{j,s'} n_{j,s'} \\
& - \sum_{s'=s}^{\infty} \sum_{\theta^{s'}} \frac{v_{i}\left(\theta^{s'}\right)}{\left[\Pi_{j=s}^{s'}\left(1-v_{i}\left(\theta^{j}\right)\right)\right]} \frac{\partial V_{i,s'}^{aut}\left(\theta^{s'}\right)}{\partial n_{i,s}} \\
& - \begin{cases} \sum_{s'=s}^{\infty} \sum_{\theta^{s'}} \frac{\mu_{-i}\left(\theta^{s'}\right)}{M_{i}\left(\theta^{s}\right)} \frac{\tilde{v}_{i}\left(\theta^{s'}\right)}{\Pi_{j=s}^{s'}\left(z\left(\theta^{j}\right)-\tilde{v}_{i}\left(\theta^{j}\right)\right)} \frac{\partial V_{-i,s'}^{aut}\left(\theta^{s'}\right)}{\partial n_{i,s}} & if\ i=1 \\ \sum_{s'=s}^{\infty} \sum_{\theta^{s'}} \frac{\mu_{-i}\left(\theta^{s'}\right)}{M_{i}\left(\theta^{s}\right)} \frac{\tilde{v}_{i}\left(\theta^{s'}\right)}{\Pi_{j=s}^{s'}\left(\frac{1}{z\left(\theta^{j}\right)}-\tilde{v}_{i}\left(\theta^{j}\right)\right)} \frac{\partial V_{-i,s'}^{aut}\left(\theta^{s'}\right)}{\partial n_{i,s}} & if\ i=2 \end{cases}
\end{aligned}
$$

Outside option

The value of the outside option for the deviating agent consists of the instantaneous value of consuming his production while free-riding on externalities-internalizing labor supply decision of the other agent in the period of deviation and later on of being in permanent autarky. The relevant state variables for the deviating household are the levels of [1] current productivity shocks; and [2] pollution at the end of previous period, i.e. $(\theta_1, \theta_2, P_{t-1}) \in \Theta \times \mathrm{R}_+$. Formally, the problem reads:

$$\begin{aligned} V_{i,t}^{nc}(\theta_1, \theta_2, P_{t-1}) &= \max_{\{\mathrm{c_{i,t}, n_{i,t}}\}} \mathrm{E}\left[\sum_{s=t}^{\infty} \beta^{s-t} u\left(c_{i,s}(\theta^s), n_{i,s}(\theta^s)\right) | \theta^t\right] \\ & s.t. \\ c_{i,s}(\theta^s) &\leq (1 - D(P_s(\theta^s)))\, \theta_{i,s} n_{i,s}(\theta^s) \;\; \forall s \geq t \\ n_{-i,t}(\theta) &= \text{argmax } (20) \\ n_{-i,t+s}(\theta) &= \text{argmax } (26), \;\; s \geq 1 \end{aligned} \tag{26}$$

[IN PROGRESS]

1. Solve the model with VFI.
2. Calibrate the dynamic model to understand the productivity and insurance gains due to the interaction of informal risk sharing with co-operation over the rice production in rural India.

7 Conclusion [see Introduction]

Motivated by the anthropological evidence on lives of traditional societies, this paper studies a dynamic model of risk sharing and common good co-operation with voluntary participation. The analysis suggests that institutions aimed at solving the problems associated with common goods may be complementary with risk sharing among the participating countries. Firstly, combining co-operation on the common good with risk sharing institutions allows for efficient usage of the common resource. In particular, the currently less productive countries reduce their production in order to prevent inefficient pollution, for which they are compensated through the risk sharing arrangements. Secondly, having a risk sharing agreement that is also co-operating on the common good's use internalizes all the externalities and so increases the aggregate amount of goods leading to a welfare and/or consumption smoothing (or risk sharing) improvement among the members of the agreement.

However, in order to benefit from this institutional mutual reinforcement, the access to the common goods shared among the risk sharing parties should be strictly regulated. If exclusion from the common good is impossible, higher efforts to maintain the common good increase the incentives of the contract's signatories to unilaterally default on their risk sharing promises. In such a case, the participation constraints become tighter and so the extent of risk sharing sustainable in the long run may actually decrease.

8 Appendix A: News evidence on change in MSP for rice

FT FINANCIAL TIMES

Companies and Markets: Commodities and Agriculture - Indian rice prices

57 words
1 September 1982
Financial Times
FTFT
Page 18
English

INDIA'S Agricultural Prices Commission has recommended a buying price of Rupees 122 a quintal for paddy during the 1982-83 season, a rise of Rs 7 in an attempt to build up government stocks at a time when output is expected to be hit by bad weather

Figure 3: Newspaper source on announcement of a change in MSP for rice in September 1982

Paddy procurement price higher by Rs. 7: Lok Sabha questions
The Times of India (1861-current); Oct 5, 1982; ProQuest Historical Newspapers: The Times of India
pg. 23

Paddy procurement price higher by Rs. 7

NEW DELHI, October 4 (UNI).

THE government has fixed the procurement price of common variety of paddy for the 1982-83 season at Rs. 122 a quintal, the agriculture minister, Rao Birendra Singh, announced in the Lok Sabha today.

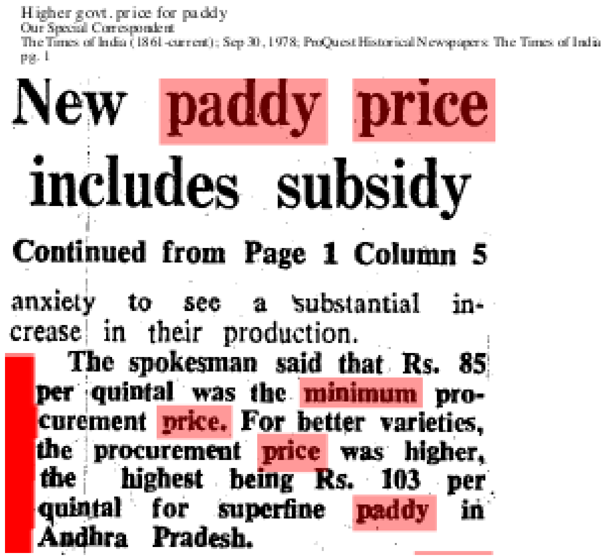
This is Rs. 7 per quintal more than that fixed for the last season.

cost of imported wheat last year, Rao Birendra Singh said.

The landed cost would include freight, port handling charges and insurance.

There were loud protests from opposition members when he told Mr. Madhu Dandavate that the government was importing wheat because procurement was not adequate to meet the needs of the expending public

Figure 4: Newspaper source on implementation of a change in MSP for rice in October 1982



Higher govt. price for paddy
Our Special Correspondent
The Times of India (1861-current); Sep 30, 1978; ProQuest Historical Newspapers: The Times of India
pg. 1

New paddy price includes subsidy

Continued from Page 1 Column 5

anxiety to see a substantial increase in their production.

The spokesman said that Rs. 85 per quintal was the minimum procurement price. For better varieties, the procurement price was higher, the highest being Rs. 103 per quintal for superfine paddy in Andhra Pradesh.

Figure 5: Newspaper source on differential MSPs depending on rice quality

9 Appendix B: Decentralizing the constrained efficient allocation

The purpose of this Section is to show that the constrained efficient allocation can be decentralized using 1) Pigouvian tax and subsidies; and 2) pollution permits in the spirit of cap-and-trade markets employed in practice (such as the EU ETS).

Alvarez and Jermann (2000) showed how to decentralize a constrained efficient allocation in Kehoe and Levine (1993) with many agents. Notice that the economy outlined here is isomorphic to theirs with log preferences. This is true due to the assumed GHH preferences implying that the labor supply decision depends only on the current realization of the shocks. Moreover, the problem studied in this paper is nicely convex. Thus, there exists a mapping of endowment shocks to productivity shocks implying equivalent net consumptions levels in autarky and in risk sharing. This similarity between the environments allows me to base my results on the work of Alvarez and Jermann (2000).

9.1 Pigouvian decentralization

I define now a competitive equilibrium with complete markets using Arrow securities, labor income tax, abatement subsidy, lump-sum transfers and solvency constraints. The solvency constraints prohibit agents from holding large amounts of contingent debt so that in equilibrium they do not want to default. These solvency constraints will be state-contingent, since the value of going to autarky varies with the state. The labor income tax and abatement subsidy are correcting the inefficient labor supply and abatement decisions. Lump-sum transfers return the difference in tax revenue and subsidy spending to the agents. Notice the the equilibrium is defined for the $N \leq \bar{N}$ number of countries co-operating on risk sharing and the extent of the common good use, in line with the planner's problem in Section 4.

Denote first by $q_t(\theta^t, \theta')$ the period t state (with its history) θ^t price of one unit of the consumption good delivered at $t+1$, contingent on the realization of $\theta_{t+1} = \theta'$, by $\tau_i(\theta^t)$ the period t labor income tax on agent i with history θ^t and by $s_i(\theta^t)$ the period t abatement subsidy for agent i with history θ^t. Accordingly, denote by $a_{i,\theta'}$ the holdings of this security by agents, and by $B_{i,t+1}(\theta^t, \theta')$ the lower limit on holdings of assets by agent i.

Definition 7.1.1: An equilibrium with solvency constraints $\{B_i\}$, labor income tax $\{\tau_i\}$, abatement subsidy $\{s_i\}$ and lump-sum transfers $\{T_i\}$ for initial conditions $\{a_{i,0}\}$ has quantities $\{c_i, a_i, n_i, x_i\}$ and prices $\{q\}$ s.t.:

(i) for each $i \in \{1, ..., N\}$, $\{c_i, a_i, n_i, x_i\}$ solves:

$$V_{i,t}(a, \theta^t) = \max_{c_i, n_i, x_i, \{a_{i,\theta'}\}_{\theta' \in \Theta}} \left\{ \log\left(c_{i,t} - \gamma n_{i,t}^{\phi}\right) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta_t) V_{i,t+1}\left(a_{i,\theta'}, (\theta', \theta^t)\right) \right\} \tag{27}$$

s.t.

$$c_{i,t} + \sum_{\theta'} q_t\left(\theta^t, \theta'\right) a_{i,\theta'} \leq a_{i,\theta} + (1 - D(Z))\,\theta_{i,t} n_{i,t}\left(1 - \tau_{i,t}\left(\theta^t\right)\right) + T_{i,t} - \left(1 - s_{i,t}\left(\theta^t\right)\right) x_{i,t} \tag{28}$$

$$(\psi) \quad a_{\theta'} \geq B_{i,t+1}\left(\theta^t, \theta'\right) \ \forall \theta' \in \Theta \tag{29}$$

(ii) markets clear:

$$\sum_i c_{i,t} = (1 - D(Z_t)) \sum_i \theta_{i,t} n_{i,t} \ \forall t, \theta^t \tag{30}$$

$$\sum_i a_{i,\theta'} = 0 \ \forall \theta' \tag{31}$$

$$T_{i,t} = (1 - D(Z_t))\,\theta_{i,t} n_{i,t} \tau_{i,t}\left(\theta^t\right) - s_{i,t}\left(\theta^t\right) x_{i,t} \tag{32}$$

Notice that in period 0 there is a one to one map between Lagrange multipliers $\{\lambda_i\}_i$ and initial wealth $\{a_{i,0}\}_i$.

Given that the objective function is concave and the constraint set is convex (as argued in Section 3.1), the associated FOC's and transversality condition are both necessary and sufficient:

$$
\begin{aligned}
a_{i,\theta'}:\ \beta\pi\left(\theta'|\theta_t\right)\frac{c_{i,t}-\gamma n_{i,t}^{\phi}}{c_{i,t+1}-\gamma n_{i,t+1}^{\phi}} &\leq q_t\left(\theta^t,\theta'\right) && (33)\\
n_i:\ \left(1-D\left(Z\right)\right)\theta_i\left(1-\tau_{i,t}\left(\theta^t\right)\right)+\frac{d\left(1-D\left(Z\right)\right)}{dn_i}\theta_i n_i\left(1-\tau_{i,t}\left(\theta^t\right)\right) &= \gamma\phi n_i^{\phi-1} && (34)\\
x_i:\ \frac{\partial\left(1-D\left(Z\right)\right)}{\partial x_i}\theta_i n_i\left(1-\tau_{i,t}\left(\theta^t\right)\right) &= 1-s_{i,t}\left(\theta_{i,t}\right) && (35)\\
\lim_{t\to\infty}\sum_{\theta^t\in\Theta^t}\beta^t\frac{1}{c_{i,t}-\gamma n_{i,t}^{\phi}}\left[a_{i,t}\left(\theta^t\right)-B_{i,t}\left(\theta^t\right)\right]\pi\left(\theta^t|\theta_0\right) &= 0 && (36)
\end{aligned}
$$

Note that FOC (27) hold with equality if $a_{i,\theta'} > B_{i,t+1}\left(\theta^t,\theta'\right)$ (i.e. agent is unconstrained).

Thus, given an equilibrium allocation $\{c_i^*, a_i^*, n_i^*, x_i^*\}$ we can now define the equilibrium one period ahead and time 0 Arrow security prices as:

$$
\begin{aligned}
q_t^*\left(\theta^t,\theta'\right) &= \max_{i\in\{1,2\}}\left\{\beta\pi\left(\theta'|\theta_t\right)\frac{c_{i,t}-\gamma n_{i,t}^{\phi}}{c_{i,t+1}-\gamma n_{i,t+1}^{\phi}}\right\} && (37)\\
Q_0^*\left(\theta^t|\theta_0\right) &= q_0^*\left(\theta_0,\theta_1\right)\cdot q_1^*\left(\theta_1,\theta_2\right)\cdot\ldots\cdot q_{t-1}^*\left(\theta^{t-1},\theta_t\right) && (38)
\end{aligned}
$$

For technical purposes, I introduce versions of three definitions/assumptions from Alvarez and Jermann (2000).

Definition 7.1.2: An equilibrium has solvency constraints that are not too tight if:

$$
V_{i,t+1}\left(B_{i,t+1}\left(\theta^t,\theta'\right),\theta^{t+1}\right)=\mathrm{E}\sum_t\beta^t\log(c_{i,t}^{aut}-\gamma n_{i,t}^{aut,\phi})\ ,\forall t\geq 0,\theta^{t+1}\in\Theta
$$

Definition 7.1.3: The implied interest rates for the allocation $\{c_i^*, n_i^*, x_i^*\}$ are high if:

$$
\sum_{t\geq 0}\sum_{\theta^t\in\Theta^t}Q_0\left(\theta^t|\theta_0\right)\left(\sum_i\left[c_{i,t}^*\left(\theta^t\right)-\gamma n_{i,t}^*\left(\theta^t\right)-x_{i,t}^*\right]\right)<+\infty
$$

.

Assumption 7.1.4: There exists a constant $\chi < +\infty$ s.t. for all t and θ^t,

$$
|u\left(\tilde{c}_{i,t}\left(\theta^t\right)\right)|\leq\chi u'\left(\tilde{c}_{i,t}\left(\theta^t\right)\right)\tilde{c}_{i,t}\left(\theta^t\right)
$$

where, as before, $\tilde{c} = c - \gamma n^{\phi}$ denotes the effective consumption. Notice also that with our log GHH utility function, where the effective consumption is bounded away from 0, this assumption is satisfied automatically.

Now I state the version of crucial results in Alvarez and Jermann (2000) adapted to the model studied in this paper. I skip the formal proofs, as given the convexity of the problem studied here, the standard separating hyperplane argument can be applied just as in their work.

Proposition 7.1.5 (Second Welfare Theorem): Suppose the constrained efficient allocation $\{c_i^*, n_i^*, x_i^*\}$ satisfies the high implied interest rates condition. Then, there exist solvency constraints $\{B_i\}$, initial wealth $a_{i,0}$ and asset holdings $\{a_i^*\}$, Pigouvian labor income tax rates $\{\tau_i\}$, Pigouvian abatement subsidies $\{s_i\}$, lump-sum transfers $\{T_i\}$ s.t. the quantities $\{c_i^*, a_i^*, n_i^*, x_i^*\}$ are a competitive equilibrium. Moreover, the solvency constraints $\{B_i\}$ can be chosen s.t. they are not too tight for all agents.

The optimal Pigouvian tax and subsidy rates have to satisfy:

$$\tau_i = \frac{-\frac{\mathrm{d}(1-D(Z))}{\mathrm{d}n_i}\sum_{-i}\theta_{-i}n_{-i}}{\theta_i\left((1-D(Z))+\frac{\mathrm{d}(1-D(Z))}{\mathrm{d}n_i}n_i\right)} > 0 \quad (39)$$

$$s_i = \frac{\mathrm{d}(1-D(Z))}{\mathrm{d}x_i}\left[\tau_i\theta_i n_i + \sum_{-i}\theta_{-i}n_{-i}\right] > 0 \quad (40)$$

Notice that the optimal tax-subsidy schedule suggests that decision on the Pigouvian tax should be complemented and co-ordinated with the decision on the subsidy, i.e. the two are inter-related. In general, in order to incentivize efficient production, the tax rate is decreasing in country's own productivity θ_i. Moreover, the subsidy for country i increases when 1) the productivity θ is high (which increases both the marginal product of abatement $\frac{\mathrm{d}(1-D(Z))}{\mathrm{d}x}$ and the overall production function efficiency); or 2) other countries are very productive and so would benefit a lot from the abatement by country i. Otherwise, the subsidy decreases in the tax rate as, by encouraging higher output, a lower tax rate stimulates higher abatement too.

Moreover, had environment been a stock variable there would only be a level change in expressions (31) and (32). In particular, if e.g. for some reason the accumulated pollution level was too high (relative to some sustainable level), the optimal tax and subsidy rate would both be somewhat increased in order to bring the environment back to its sustainable level.

Now, I state the version of Alvarez and Jermann's (2000) First Welfare Theorem.

Proposition 7.1.7 (First Welfare Theorem): If assumption 5.1.4 is satisfied, an equilibrium with solvency constraints that are not too tight, with high implied interest rates, with the optimal Pigouvian tax-subsidy schedule and lump-sum transfers clearing the markets, is efficient.

Propositions 7.1.5 and 7.1.7 confirm that we can actually decentralize the constrained-efficient allocation using the Pigouvian instruments. In the remaining decentralizations below, I do not repeat all the theory from this Section as it involves only making some straightforward adjustments following directly from the definitions of competitive equilibria considered.

9.2 Cap-and-trade decentralization

Now, I show that we can also decentralize the constrained efficient allocation using pollution permits in the spirit of cap-and-trade markets employed in practice (such as the EU ETS). I first define the competitive equilibrium with pollution permits, Arrow securities, solvency constraints and lower bounds on the pollution permit holdings. The latter ensures that the amount of pollution rights purchased by the agent is actually covering his pollution net of abatement.

Definition 7.2.1: An equilibrium with solvency constraints $\{B_i\}$, pollution permit holding constraint $\{Z_{i,t}\}$, optimal cap on the aggregate amount of pollution permits in the markets $\bar{Z}(\theta^t)$, for initial conditions $\{a_{i,0}\}$ has quantities $\{c_i, a_i, \tilde{a}_i, n_i, x_i\}$ and prices $\{q, \tilde{q}\}$ s.t.:

(i) for each $i \in \{1, ..., N\}$, $\{c_i, a_i, \tilde{a}_i, n_i, x_i\}$ solve:

$$V_{i,t}\left(a_{i,\theta^t}, \theta^t\right) = \max_{c_i, n_i, x_i, \{a_{i,\theta'}\}_{\theta' \in \Theta}, \tilde{a}_{i,\theta^t}} \left\{ \log\left(c_{i,t} - \gamma n_{i,t}^{\phi}\right) + \beta \sum_{\theta' \in \Theta} \pi\left(\theta' | \theta_t\right) V_{i,t+1}\left(a_{i,\theta'}, \theta'_i, \theta'_{-i}\right) \right\} \tag{41}$$

s.t.

$$c_{i,t} + x_{i,t} + \sum_{\theta'} q_t\left(\theta^t, \theta'\right) a_{i,\theta'} + \tilde{q}_t\left(\theta^t\right) \tilde{a}_{i,\theta_t} \leq a_{i,\theta^t} + \left(1 - D\left(Z_t\right)\right) \theta_i^t n_{i,t} \tag{42}$$

$$(\phi) \quad \tilde{a}_{i,\theta_t} \geq Z_{i,t} \tag{43}$$

$$(\psi) \quad a_{i,\theta'} \geq B_{i,t+1}\left(\theta^t, \theta'\right) \ \forall \theta' \in \Theta \tag{44}$$

(ii) markets clear:

$$\sum_i c_{i,t} = \left(1 - D\left(Z_t\right)\right) \sum_i \theta_{i,t} n_{i,t} \ \forall t, \theta^t \tag{45}$$

$$\sum_i a_{i,\theta'} = 0 \ \forall \theta' \tag{46}$$

$$\sum_i \tilde{a}_{i,\theta^t} = \bar{Z}\left(\theta_t\right) \ \forall \theta_t \tag{47}$$

Importantly, $\bar{Z}(\theta_t) = \sum_{i=1}^{N} Z_{i,t}(\theta_t)$ with $Z_{i,t}(\theta_t)$ being the equilibrium net pollution quantities from the planners problem (12) for the N co-operating countries.

Furthermore, this decentralization can be also seen as the Lindahl (1956) equilibrium extended by risk sharing: agents 'consume' the same amount of the common good $(1 - D(Z))$ but they may pay different price for its provision. In particular, an agent with high productivity who is producing a lot is going to buy more of the pollution permits and so contribute more to the common good.

Given that the objective function is concave and the constraint set is convex (as argued in Section 3.1), the associated FOC's are both necessary and sufficient:

$$a_{i,\theta'}: \ \beta\pi(\theta'|\theta_t)\frac{c_{i,t} - \gamma n_{i,t}^{\phi}}{c_{i,t+1} - \gamma n_{i,t+1}^{\phi}} \leq q_t(\theta^t, \theta') \tag{48}$$

$$\tilde{a}_{i,\theta^t}: \ \tilde{q}_t(\theta^t)\frac{1}{c_{i,t} - \gamma n_{i,t}^{\phi}} \leq \phi \tag{49}$$

$$n_i: \ (1 - D(Z))\theta_i + \frac{\partial(1 - D(Z))}{\partial n_i}\theta_i n_i = \gamma\phi n_i^{\phi-1} \tag{50}$$

$$x_i: \ \frac{\partial(1 - D(Z))}{\partial x_i}\theta_i n_i = 1 \tag{51}$$

$$\lim_{t\to\infty}\sum_{\theta^t\in\Theta^t}\beta^t\frac{1}{c_{i,t} - \gamma n_{i,t}^{\phi}}\left[a_{i,t}(\theta^t) - B_{i,t}(\theta^t)\right]\pi(\theta^t|\theta_0) = 0 \tag{52}$$

Notice that condition (42) will always hold with equality as by construction the pollution permits are perishable, i.e. there is no reason for agents to save the pollution permits as they become worthless in the next period. Thus, we do not need any additional transversality condition for pollution permits.

Importantly, as the average productivity in the economy increases (i.e. the economy is expanding), so does the average extent of production given by the labor supply decision n. Thus, by assumption the total cap on emissions increases as well

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